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Evolution of Non-Institutional Representations in a Modeling Process of Teaching (ACODESA)

Evolución de Representaciones No Institucionales en un Proceso de Modelación en la Enseñanza (ACODESA)

Fernando Hitt Espinoza @ [1], Samantha Quiroz Rivera @ [2]

[1] Cinvestav-UQAM (México)
[2] Universidad Autónoma de Coahuila (México)

Abstract ∞ The purpose of the study is to analyze the process of mathematical knowledge co-construction from a sociocultural perspective. Using a qualitative methodology, we present the results of implementing five research situations and how these results relate to the covariation between variables, this as a prelude to the concept of function. Using a multiple case study design, we present the results for four high school students aged 14 to 15. The results focus on the production process of functional-spontaneous representations and their evolution towards Socially constructed representations. This evolution process is determined by the discussion of ideas across the five stages of the ACODESA method. We also discuss the concept of *habitus* in the mathematics classroom. The study concludes that Socially constructed representations in mathematics classrooms are both constructed and transformed through a process of communication and objectification of signs or concepts.

Keywords ∞ Conceptual imagery; Mathematical model; Secondary education; Social learning; Teaching method

Resumen ∞ El propósito del presente estudio es analizar el proceso de construcción del conocimiento matemático desde una perspectiva sociocultural. Mediante una metodología cualitativa, se muestran los resultados de la implementación de cinco Situaciones de Investigación y cómo se relacionan con la covariación entre variables, como un preludio del concepto de función. Se diseña un estudio de casos múltiples; se presentan los resultados de cuatro alumnos de escuela secundaria cuyas edades oscilan entre los 14 y15 años. Los resultados se enfocan en el proceso de producción de Representaciones Funcionales Espontáneas y cómo estas evolucionan hacia Representaciones Socialmente Construidas. Este proceso de evolución es determinado por las cinco etapas del método ACODESA. También se discute el concepto de *habitus* en el salón de matemáticas. El estudio concluye que las Representaciones Socialmente Construidas en el salón de matemáticas emergen y se transofrman en un proceso de comunicación y objetivación del signo o concepto.

Palabras clave ∞ Aprendizaje social; Enseñanza secundaria; Método de enseñanza; Modelo matemático; Representacion mental

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1. Introduction

In the last century, following the constructivist theoretical framework of learning, a great body of research was conducted on the learning of mathematics. However, the complexity of analyzing the classroom as a micro society brought along many efforts to understand the implications involved in the co-construction of mathematical knowledge within the classroom (D'Ambrosio, 2000; Hitt, 2013; Radford, 2003; Yackel & Cobb, 1996). At the beginning, researchers thought that the change of paradigm would lead to a natural transition from constructivism to socio-constructivism. Nevertheless, von Glasersfeld (2001), a radical constructivist, affirms:

> Social constructivism is a recent movement initiated by people who claim that radical constructivism ignores the role of social interactions in the construction of knowledge. This statement, as I understand it, is justified in part by the fact that until now neither Piaget nor the most recent constructivists have proposed a detailed model of the functioning of social interaction in the same constructivist perspective. However, Piaget and all those who were inspired by his work have always maintained that social interaction is very important in the construction of knowledge. But first, we had to develop models for all the elementary constructions that must be made by a cognitive organism before we can begin to know and interact with others. On the other hand, as far as I know, the social "constructivist" tends to consider society as given, and a radical constructivist cannot accept that. In my opinion, society must be analyzed as a constructed concept if we are going to correctly explain and assess its role in the subsequent construction of concepts. (p. 294, translation by the authors)

When we talk about learning as a socially constructed process, questions such as how can we characterize the elemental constructions in a group in a social learning environment through collaboration emerge. Answering implies taking an epistemological stance that facilitates the explanation of how elemental constructions are developed within a social environment.

The present study adds to this effort and opts for the choice of a theoretical framework related to the construction of knowledge under a sociocultural approach (Hitt, 2013). The objective is to integrate the contributions of the sociocultural school represented by Cole (1996), Engeström (1999), Leontiev (1978), Voloshinov (1973), and Vygotsky (1962) as theoretical bases and reflections, following this line of thought, within mathematics education (Radford, 2003).

Specifically, this study focuses on the notion of covariation between variables as a prelude to the concept of function. The contributions of the study not only strengthen the initial theoretical bases, but they also raise more questions to carry out further studies on this line of research, as was pointed out by von Glasersfeld (2001).

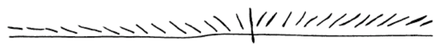
2. A Sociocultural Theoretical Framework in the Learning of Mathematics

In this section, we attempted to describe the theoretical foundations that guide our work. This supports the research question to be answered in the following sections.

2.1. Research Situations and Functional-Spontaneous Representations

Curricular changes have established mathematical competence as a key objective. To foster it, it is essential to define the classroom tasks proposed by teachers, often categorized as exercises, problems, or open problems (see Table 1).

Table 1. Definition of Exercise, Problem, and Open Problem

	Definition	Example
Exercise	When a mathematical statement refers to a procedure already established by the individual (Soto et al., 2019).	Solve the equation system: $x + y + z = 0$; $3x + y - 2z = 1$; $6x - 2y + 4z = 1$
Problem	A statement that does not promote in the reader a reference to a certain procedure, and that forces it to the construction of a particular internal representation to relate different representations and strategies of what is at stake, which promotes the articulation between different representations (Soto et al., 2019).	In Euclidean Geometry, is there a triangle in which the measures of their angles α, β, γ satisfy the conditions: $\alpha + \beta = 100°$; $\beta + \gamma = 100º$? (Hidden information $\alpha + \beta + \gamma = 180°$).
Situation	An open activity to promote the emergence of spontaneous representations in the resolution process (diSessa et al., 1991, p. 122).	"A motorist is speeding across the desert, and he's very thirsty. When he sees a cactus, he stops to get a drink from it. Then he gets back in his car and drives slowly away."  Mitchel explained: "If the line is horizontal, he's going really fast. And the further up the line slants, the slower it goes. And then when it gets like this (vertical), it stops" (diSessa et al., 1991).
Model-eliciting activities	"The situation to be understood involves some type of mathematically interesting system which often (but not always) exists outside the world of mathematics. So the most problematic aspects of tasks often involve developing useful ways to think about (describe, explain, interpret) relevant relationships, patterns and regularities or givens, goals, and possible solution paths so that relevant mathematical tools can be used." (Lesh & Yoon, 2007, pp. 166-167).	The local police detective discovered that some people rebuilt the old brick drinking fountain in the park, and he would like to thank the people who did it. All the police could find were lots of footprints. One of the footprints is shown here. But to find this person and his/her friends, it would help if we could figure out how tall he or she really is. Your job is to make a "HOW TO" TOOL KIT that the police can use to figure out how tall people are just by looking at their footprints (Koellner-Clark & Lesh, 2003).

A common characteristic found in exercises and closed problems is their focus on institutional representations. In contrast, open situations such as those mentioned by diSessa et al. (1991) and model-eliciting activities (Lesh & Yoon, 2007) stimulate divergent thinking in students who can be challenging to manage.

For this purpose, we propose the implementation of research situations (RS), defined as the set of interconnected tasks necessary to address challenges within a sociocultural and technological learning environment. This approach promotes the formation of spontaneously arising representations in the classroom. We consider it necessary to set aside theoretical tools that only privilege the use of institutional representations (Duval, 1995; Janvier, 1987).

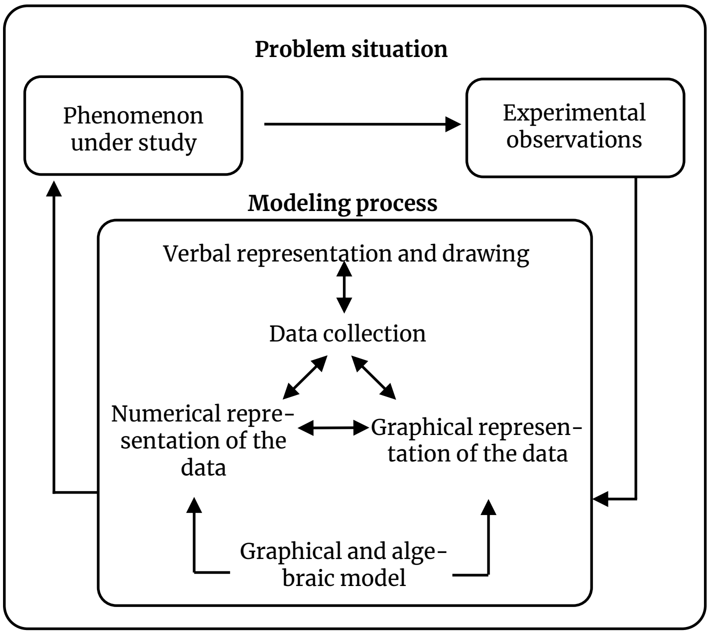
In our research, we have defined *functional-spontaneous representations* (F-SR) as representations that emerge in the individuals during practice when facing a non-ordinary activity. The actions related to the interaction with the activity within the context have functional cognitive characteristics and are linked to an (external) spontaneous representation. The representation is functional in the sense that it emerges from a specific motivation or reason to provide meaning to the individual as the activity is being performed, and it is spontaneous since it arises naturally in action while attempting to solve a non-ordinary situation or RS (Hitt & Quiroz, 2017).

The design of the RS follows a process in which students are expected to use or create mathematical models from a specific situation. The term model is taken from the work of Lesh & Doerr (2003), who define it as:

> A model is a system consisting of (a) elements, (b) relationships among elements, (c) operations that describe how the elements interact, and (d) patterns or rules, such as symmetry, commutativity, or transitivity, that apply to the preceding relationships and operations. (p. 362)

To respond to the RS, the modeling process involves transitioning through different representations, with verbal and drawing representations standing out as intimately related to the F-SR. In general, we aimed to follow the diagram presented in Figure 1.

Figure 1. Model of the RS, Linked to the Mathematical Modeling Process



2.2. ACODESA as a Teaching Method

The ACODESA teaching method (Collaborative Learning, Scientific Debate, Self-reflection) emphasizes the construction of knowledge from a sociocultural perspective (Hitt, 2013). The assignment begins with an initial approach to an RS, which will be solved in the classroom through five class organization strategies (see Figure 2).

2.2.1. Individual Work

After reading the RS, the goal is to provoke divergent thinking in students, promoting the production of the F-SR. This prepares students for discussions with members of their team (Hitt & González-Martín, 2015). At this stage, students are better prepared to communicate more efficiently to validate their own proposals, as well as those of their classmates.

2.2.2. Teamwork

During teamwork, students are encouraged to discuss and build a new representation that incorporates the ideas discussed among their peers. Voloshinov's work (1973) emphasizes communication in the construction of the sign. As Eco (1975/1992) and Radford (2003) suggest, our interest is the process of objectification that occurs in the continuous communication process leading to the construction of the sign.

2.2.3. Group Discussion

At this stage, communication and validation are encouraged, but this time through a scientific debate involving the whole group (in terms of Legrand, 2001; Hitt & González-Martín, 2015). This process of objectification leads precisely to the establishment of new representations and the refinement of those previously produced.

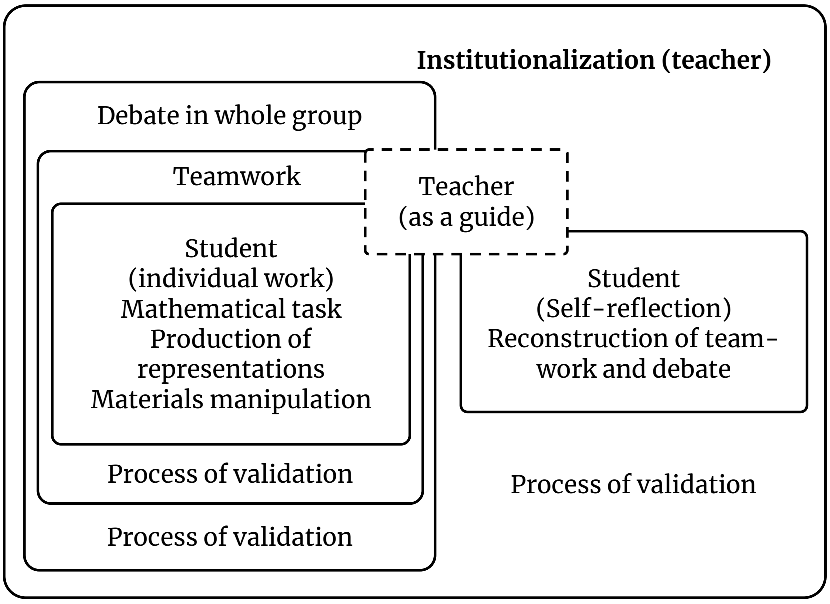
2.2.4. Self-Reflection

This stage aims to reconstruct the work done in the previous stages. Given that consensus in the mathematics classroom is temporary and fragile (Hitt & González-Martín, 2015; Karsenty, 2003; Thompson, 2002), it is crucial to reconstruct the results obtained from previous stages.

2.2.5. The Process of Institutionalization

As a final stage, the ACODESA teaching method proposes that the teacher organize their presentation by considering the production and evolution that occurred in class during the previous stages, prior to providing the official representation.

Figure 2. The Teacher as a Guide in the Early Stages of ACODESA



2.3. Co-construction of Mathematical Knowledge in a Sociocultural Environment

Voloshinov's studies (1973) focus on the role of language and communication in the construction of ideological signs: "The reality of the sign is wholly a matter determined by that communication. After all, the existence of the sign is nothing but the materialization of that communication" (p. 13).

Vygotsky (1962) references the work of Voloshinov by stating that language not only accompanies the activity but also serves as an instrument for thought that directs, seeks, and intentionally plans the individual's actions. Interpersonal communication plays a decisive role in the formation of concepts through a creative process.

Leontiev's studies on activity introduce two concepts: action and operation. He emphasizes that actions are not their only point of interest, but also their cognitive component: the operation. Thus, actions can be both mental and physical, while operations are mental and incorporated into other senses. In this context, the functional-spontaneous representation would be viewed as indivisible, with its internal and external components linked as a unit.

3. Research Question and Methodological Aspects

The theoretical elements presented in the previous section provide us with a foundation for the research question that will guide our study: How is knowledge constructed within a sociocultural environment in the mathematics classroom? To answer this question, we will analyze three main elements: the formation of Spontaneous Functional Representations, their evolution in communication processes following the ACODESA method of teaching, and the process of constructing the notion of covariation between variables.

To this end, the present investigation adopted a qualitative approach (Hernández et al., 2010). Specifically, a Multiple Case Study design has been chosen since more than one case will be selected (Creswell, 2007). The study population consisted of two groups of third-grade students, aged 14 to 15, from a secondary school in the city of Montreal, Canada. From these groups, four students (two girls and two boys, who formed a team) were chosen as the main sample for analysis. The selection of these students was not based on statistical logic but sought to reach a level of comparability by exhaustively examining the characteristics of the phenomenon that reinforce the validity of the conclusions. For confidentiality purposes, they will be referred to as C1, C2, C3, and C4.

Five research situations were designed to promote covariation between variables. The first RS aimed to develop only verbal representations and diagrams, as it was presented in an introductory manner. The second situation incorporated into its objectives the production of graphical representations. However, it was only in RS 3, 4, and 5 that students were expected to establish algebraic representations (see Table 2).

Table 2. Organization of Activities and Type of Representation

Research situation	Name	Type of representation promoted			
		Verbal	Diagram	Graphic	Algebraic
RS1	The photographer	√	√	X	X
RS2	The hiker	√	√	√	X
RS3	The jacuzzi	√	√	√	√
RS4	The squares	√	√	√	√
RS5	The shadows	√	√	√	√

The total implementation consisted of 13 lessons, each lasting 70 minutes. Video recordings were made to collect data during video sessions. The materials produced by the students were also analyzed, and some informal interviews were conducted during the resolution and teamwork sessions. The purpose of using different data collection techniques is to ensure construct validity (Yin, 2014).

The analysis of the video recordings was based on the model proposed by Powell et al. (2003), which outlines a sequence of steps: carefully review the video data, describe the content thereof, identify critical events, transcribe and encode the data, build a sequence of events, and write a narrative.

4. Analysis of Results

We will present the results using three of the research situations. Table 3 shows the rationale for the selection of each Research Situation.

Table 3. Criteria of Analysis of Each RS

RS	Opening summary	Objective of election	Stage of ACODESA analyzed
RS1	A professional photographer walks near the statue of Jacques Cartier. He takes pictures from different places on the sidewalk. For each picture he takes, he notes the place where he stands on the sidewalk and the distance that separates him from the statue. Once the photos are ready, he can choose the places where he should be placed to take the best photos. For each point where the photographer is positioned, the distance between him and the statue varies. Describe the phenomenon with your words and draw a picture of the situation to illustrate.	Emergence of the F-SR	Individual work
RS2	A hiker undertakes a long excursion in a forest. He follows a closed path. The track never passes by the same point twice, completing only one circuit. An aid station is located within the area surrounded by the track. A flagpole makes it possible for the hiker to locate the site of the aid station. Draw a track of your choice and place the security post inside. The distance between the hiker and the security post varies according to the place where the hiker is on the track. Describe this variation.	Process of evolution of said representations	Individual work Teamwork Whole group discussion
RS5	Suppose we have a light source at a height of 6 meters; we can see a shadow on the ground since a person of 1.5 m height walks on the street. We are interested in the relationships of the measures at stake. Are there measures that are dependent on each other? Which? Select two measures that are dependent on each other and describe the phenomenon with the help of different representations that you used in the previous activities.	Covariation between variables and functional relationship	Self-reflection

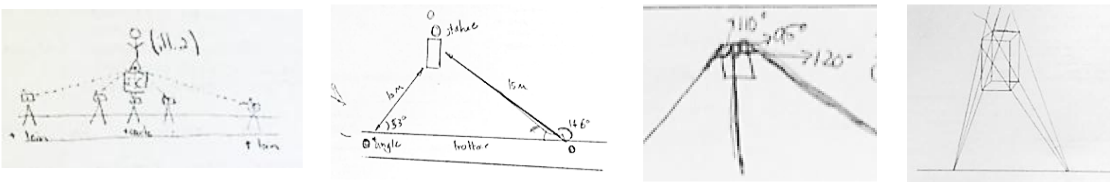
4.1. RS1 “The Photographer”

In this initial situation, the variables related were the following: the distance between the camera and the statue and the distance between the camera and a reference point on the sidewalk.

4.1.1. Individual Work

After the initial presentation, the students worked individually, where they were asked to describe the phenomenon in their own words and through a drawing. These initial representations emerged spontaneously and were intended to answer the posed question, which is why they are considered F-SR. Figure 3 displays the F-SR created by the four students.

Figure 3. F-SR of the Students in “The Photographer”



C1 C2 C3 C4

It is possible to note that the four representations present some important characteristics. All four students identified elements involved in the situation—the camera, the statue, and the sidewalk—and depicted them in a realistic way. Additionally, the students established relationships between these elements, as shown by the lines drawn from the camera to the statue (C3). Another characteristic is the inclusion of mathematical elements, such as angles and distance measurements. Moreover, in the F-SR of C2, C3, and C4, the camera is represented not by a realistic drawing but by means of a point. These elements allow us to observe progress in the students’ involvement in the mathematization process.

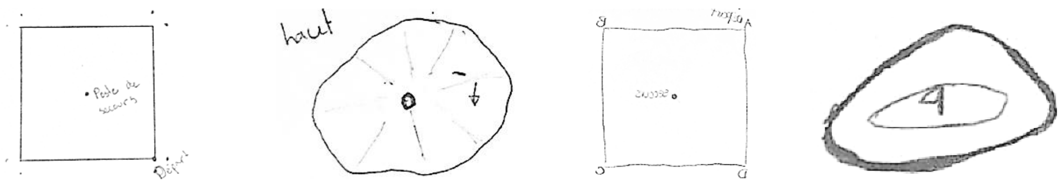
4.2. RS2 “The Hiker”

In this situation, the related variables were the following: the distance between the hiker and the security point, and the distance traveled by the hiker.

4.2.1. Individual Work

Considering the characteristics of the RS referring to the initial activity, the students were asked to draw the shape of a track that the hiker could walk through. Figure 4 shows that two students thought of a square (C1 and C3), and the other two proposed an irregular track (C2 and C4).

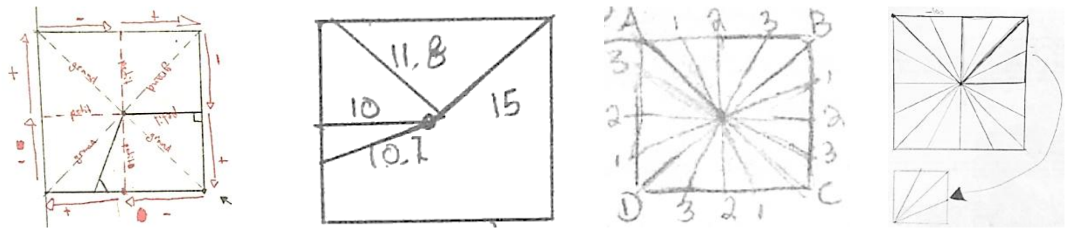
Figure 4. Tracks Proposed by C1, C2, C3, and C4, Respectively



C1 C2 C3 C4

Continuing with the individual work, the teacher made an intervention to guide the students to reach a consensus on the shape of the track. C3 suggested a square track, which was accepted by the entire class. Following this, the students individually reconstructed their initial representations (see Figure 5).

Figure 5. Square Tracks that Draw C1, C2, C3, and C4



C1 C2 C3 C4

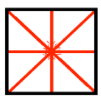
The F-SRs produced by the students incorporated more mathematical elements and relations between them. A key difference between them is the vision of the phenomenon as continuous or discrete, which generated a first discussion among the team.

In C1's representation, the student uses both signs (+) and (-) around the square that represents the track. Additionally, the words "large" and "small" are placed with arrows. This means that when the hiker is located at the corner, there is a larger distance, but as they walk along the track, this distance gradually decreases as they reach the center. After that, the (+) sign indicates that the distance becomes larger again as they reach the other corner. In their representation's description, C1 stated, "If we take the corners as a reference point, we can calculate the angles at which the walker sees the flagpole." This suggests that C1 viewed the phenomenon as a continuous process. In contrast, C2, C3, and C4 saw the phenomenon as discrete. This is evident when C3 described the distance as changing "from large to small, then large, then small, and so on."

4.2.2. Teamwork

To start the teamwork, the teacher intervened, suggesting the use of material (wooden sticks, rope, glue, wires) to represent the phenomenon. We can notice three specific moments for the use of the material presented in Table 4.

Table 4. Instances of Use of the Material

Moments	Description	Discussion
1. The wire and the ruler	C1 and C3 selected a wire to model the track. The proposal was the following:  C3: Ok, let's say the center is there [she puts her finger in the middle of the track], and we open the track [she separates the wire so that the track is like a straight line]: T: Is it the same track? C1: No T: It is not the same way, but is it the same route? C1: Yes. T: In length, yes. Then there [showing the wire], why? It represents... C3: No, it's not the same thing because here [the end of the thread] is longer than when it's folded	C3 did not agree with the use of the material. The teacher intervened: T: So, tell me, am I really interested in the security post? In the text, it is written that I am interested in what? Do we see absolutely the security post? C3: Yes T: No, it is not the security post that I see, is it what? C1: It is the distance between the hiker and the security post. Well, it's the displacement; it's the distance between him and... T: ...and why did you represent the distance between the hiker and the post... T: Why don't they use a rope instead of a ruler?
2. The use of ropes	Proposal two was carried out considering the distances between the corners and the security station and the intermediate points of the track. C1: Find the thread. C3: Listen, listen, listen! I have an idea. Let's stick this [the thread] on the... C4: Yes! C2: No, you will not be able to tie them after C1: And if we stick them? C4: Yes, because if we stick them, we will be able to redo the square, undo the square, and everything will remain the same.	For C4, the discrete idea was changed little by little during the action, now she began to describe the phenomenon as dynamic. The teacher, seeing the difficulties with the thread, put wooden sticks on the table.
3. The use of toothpicks	C4 finished by making a construction with wires and wooden sticks where it was possible to open and close the square. The use of materials caused C4 to get involved in the activity, giving rise to what Brousseau (1983) has called a "dévolution" (appropriating herself of the situation).	C4: It's the two girls who work, and the men do nothing. It's like in real life every day too. C1: It's not true C4: In the future, I will be an architect. She proposes the following assembly: 

At this stage, the students analyzed C4's creation, which displayed the track with the pieces pasted in the following manner (see Figure 6).

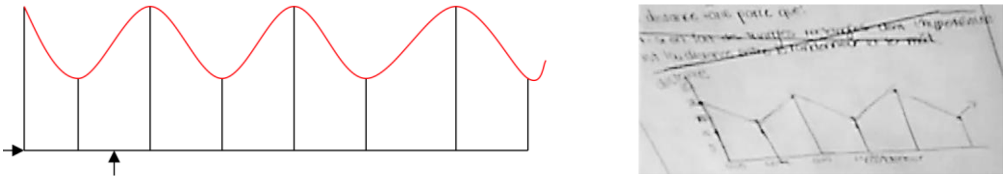
Figure 6. Construction of C4, C3 Presents the Possibility of Unfolding the Track

The students started discussing the lengths of the sticks along the track, and C1 described the phenomenon: "The more the hiker moves towards the center of a segment of the road, the distance between him and the aid station decreases, but after passing the center and approaching a corner, the distance increases." This statement allows us to reinforce the concepts of continuity and dynamism.

The following dialogue captures the moment of interaction when the students conceptualized the unfolded track as a Cartesian plane:

> C3: Look here, he walks, and the more he goes on walking, the more he gets smaller [shows the distance between the hiker and the aid station].
>
> C1: No, your starting point is here [on the lower left corner of the square]. He approaches the center of the segment [he talks about the midpoint of the base], then the distance between him and the aid station decreases, but as he passes the center, the distance increases again [lower right corner], and as it continues... Then we see that between here [the middle of the base] and here [the lower right corner], if we make a straight line and represent just these two segments here, we can see that there is a large, small, large, small alternation... [repeats in the way that C3 had expressed].
>
> C2: It was C3 who did the work, and we took the ideas.
>
> C1: Ok. Then we place the starting point here. There [the extended track] is the axis of the abscissas (1), and there, the axis of the ordinates that make a Cartesian plane. And we make like a big thread that can show a regularity in the movement [makes a mimic of the curve in red] (see Figure 7). Then the hiker...
>
> C3: Then this [extended track] are the x ...
>
> C1: Yes, this [extended track] is going to be the distance of the road, and this [makes a vertical movement] this is the distance between it and the center. Then the more he advances, it decreases, it increases, it decreases, it increases ...

Figure 7. The Gestures of C1 (Left) With the Hand and the Interpretation of C3 on Paper



The process of verbal and gestural communication, manipulation of materials, argumentation, and validation made it possible to reach this point, which was the final part of the team discussion stage.

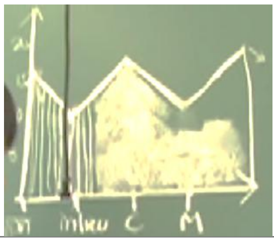
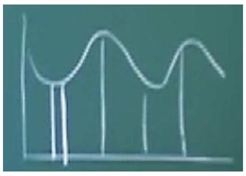
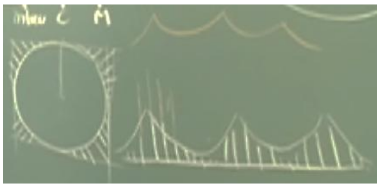
4.2.3. Whole-Group Discussion

The teacher asked the team to show their production to the whole group. C3 presented the work done with physical objects, and C2 made a graph on the blackboard (see Figure 8).

Figure 8. C3 Mentioned that the Graph of C2 is Proportional to the Wire

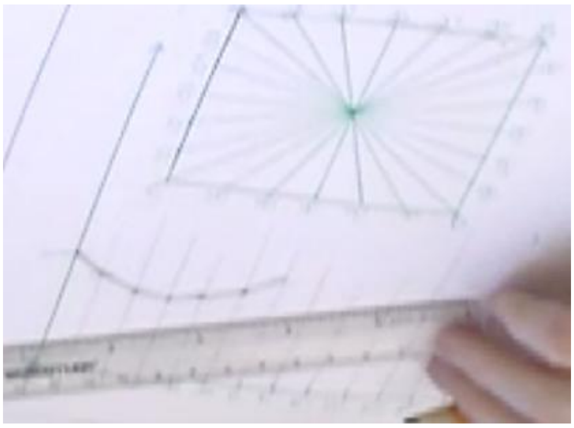
At this moment, the following discussions began in the class.

Table 5. Discussion About the Shape of the Graph

Discussion	Team 1	Team 2	Team 3
About the shape of the graph	They argued that the line was continuous and that there was an infinity of vertical lines. 	The second team proposed the following track: 	A student from Team 3 mentioned that she imagined a circle from which sticks came out, and when the circle was opened, that's when the graph was obtained. 

At this very moment of the discussion, the teacher decided to ask them to return to the teamwork by asking, "How could you determine which of the three propositions is correct?" In response, the teams suggested that additional intermediate points for the hiker should be considered. They implemented this suggestion, and their results are shown in Figure 9.

Figure 9. Representation After the Large Group Discussion Held by the Students

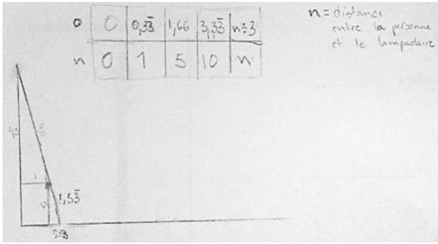
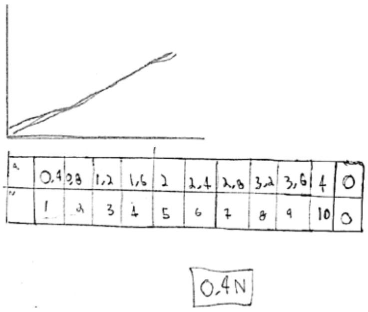
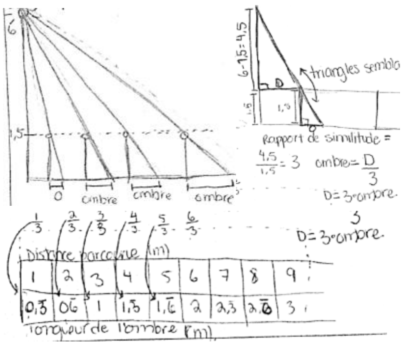
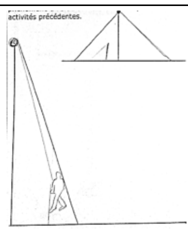


4.3. RS5 "The Shadows"

4.3.1. Self-Reflection

The representations that the students developed during the self-reflection phase are presented in Table 6. The discussion of the table highlights how each student constructed the notion of variable and the covariation between variables after working on RS5.

Table 6. Construction of the Notion of Covariation and Functional Relationship

Cases	Description	Representation
C1	C1 easily identified variables and actively contributed to establishing functional relationships, engaging attentively with peers. He expressed relationships clearly, including algebraic forms, and progressed beyond classroom discussions, aligning with Perkins and Simmons's (1988) inquiry structure. Overall, he consistently maintained global coherence across verbal, numerical, graphical, and algebraic representations.	 He placed the functional relation by means of an algebraic expression and added a table of values.
C2	He wrote little, and seemed to detect different variables but did not strive in discussions with his peers. C2 did not worry about the overall coherence of his representations.	 He showed a numerical approximation
C3	She was able to detect different variables. She managed to express very well what was discussed among the group. In the shadow problem, the functional relation: $The\ shadow = \frac{distance\ traveled}{3}$ was discovered by several teams in the two groups in a numerical way. C3 in the discussion among the large group was the one who suggested the idea of similar triangles.	 She explained without any difficulty the elements that emerged in the discussion.
C4	She made only one drawing without advancing in what was discussed by her peers. We can say that unlike C2, she probably did not understand what was being discussed.	 Her representations did not show too many elements to assess her understanding of the phenomenon.

5. Towards the Co-construction of a *Habitus* in Relation to Mathematical Modeling

We present some specific characteristics shown by the students in the development of the activities.

5.1. Designation of the Group Leader and Each Team Member's Role During Implementation

C3 initially took the lead, but this role became increasingly tenuous over time. The team understood that her ideas were most effective when discussed within them. It was observed that the students remained curious about the situation and allowed themselves to think of questions regarding the problem they would solve collaboratively (Imm & Lorber, 2013).

5.2. Validation Process of Ideas in the Team Through Communication

As the RS progressed, it became clear that the validation of ideas relied on consensual discussion among the team members. Although C2 and C4 did not show initiative in generating ideas at the beginning of the class, their subsequent interventions were crucial in obtaining the desired result. This result supports studies, which establish that collaborative learning in the classroom encourages the integration of efforts, interests, talents, and competencies to achieve collectively defined goals. (Alsina, 2007; Ikeda, 2007).

5.3. Distribution of Work According to the Abilities of Each Team Member

It was possible to observe that the actions carried out by the students reflected the abilities of each team member. While C3 proposed clear initial ideas, C1 tended to mathematize and summarize the process through algebraic and graphical representations. C4 took on a more expert role during moments involving handling material, and C2 sought to be the one who communicated ideas to the entire group. The interaction and exchange of ideas and experiences were fostered by the collaborative methodology employed. This approach also involved the selection of problems of the students' interest and their active participation in discussions and reflections (Cerqueira, 2009; Lombardo & Jacobini, 2009).

5.4. Creation of F-SR

As they experienced the RS, it became clear to the students that their initial ideas would be valued, and this motivated them to create F-SRs by incorporating additional elements that could support the resolution of the activity.

5.5. Search for Conversions Between Representations

In the last RS, students' productions showed a tendency to modify the initial representations by incorporating algebraic and graphic elements, as well as numerical tables. The students became increasingly familiar with the process of converting from one representation to another, and, at times, they did it during their individual work.

All these processes of objectification throughout the RS allow us to explore an important notion discussed by Bourdieu (1980) regarding the formation of *habitus*:

> The conditionings associated with a particular class of conditions of existence produce *habitus*, which are systems of durable and transferable dispositions,

> structured structures predisposed to function as structuring structures, as principles which generate and organize practices and representations that can be objectively adapted to their purpose without assuming the conscious aim of the purposes or an explicit mastering of the operations necessary to achieve them. (pp. 88-89, translation by the authors)

This idea of *habitus*, alongside Cole's (2000) idea that "culture 'deforms' action and brings into being of 'cultural habit of behavior'" (p. 331), can be linked to Fischbein's (1987) notion that the students' mathematical intuitions can be educated. Together, they allow us to define the notion of *habitus* in the mathematics classroom, viewed as a micro society:

5.5.1. Definition of Socially Constructed Representations and Habitus in the Mathematics Classroom

In a sociocultural approach to classroom learning, the design of interconnected activities with respect to a mathematical concept will "deform" students' actions, transforming their intuitive ideas related to their F-SR through a process involving individual work, teamwork, group discussions, and self-reflection, leading to more refined representations. We refer to these as socially constructed representations (SCR). Over time, they will be integrated when discussing institutional representations, creating a *habitus* related to the mathematical concept at play.

Table 7. General Students' Performance in the Construction of a *Habitus* in Math Class

Case	Characterization
C1	He followed and intervened in the propositions of his peers. He used to express his ideas in a way that was more closely related to the mathematical modeling than his peers. We consider that the *habitus* linked to mathematical modeling. It was fundamental to their interaction and the manipulation of materials for this co-construction.
C2	He kept up with the work of his peers. But in the processes of self-reflection, he did not advance beyond the team's collective work. The cognitive structure linked to the *habitus* did not develop as expected; he just followed his peers.
C3	She became the leader of the group. However, she could not advance in the RS activities alone. She had a need of her teammates to go further. Her focus was on understanding and expressing her ideas as best as possible. We can say that a *habitus* was developed as expected.
C4	She showed great interest in manipulating objects, with her construction in the hiker situation being crucial. She followed her teammates' ideas, but she intervened minimally. During the self-reflection phase, she struggled to reproduce what had been discussed in class. As in the case of C2, the *habitus* did not develop as expected.

6. Conclusions

6.1. Group Representations in the Mathematics Classroom

Through analysis, it was possible to observe that the F-SRs of the students in the first stage of the ACODESA method differed from those at the end of the activity. The evolution of the F-SR occurred through multiple interaction processes both in teams and in large groups, as well as during the final phase of individual reflection (self-reflection about reconstruction). New representations emerged as the result of collaborative efforts between peers and the teacher, who consistently guided the group towards an objective.

We return to the concept of *socially constructed representations in the mathematics classroom* to refer to the representations developed by students through a *collective history* in the classroom. These representations are *constructed and transformed* through a process of *objectification and communication, where signs and concepts are constructed*. This process involves individual work, teamwork, large group debates, and self-reflection.

6.2. Covariation Between Variables and Functional Relationship

In our study, the organization of the RS facilitated reflection from the very beginning on verbal representation through initial questions, followed by numerical and graphic sketching, and ultimately requesting algebraic representations. Based on the study's results, we identified several cognitive levels related to the construction of the concept of functions. These levels emerge supported by a collaborative learning approach, as presented in Table 8.

Table 8. Stages in the Construction of Variation and Generalization, Covariation Between Variables, and Functional Relationship

Content	Mathematical activity	Activity, action, operations
Discrete variable	Generalization of discrete variables (Radford, 2003; Warren, 2006).	Given a series of objects, verbalization of prediction of the characteristics of an object in the "nth-element".
Continuous variable	Generalization of continuous variables (diSessa et al., 1991).	Verbalization of the variable time, the variable volume when filling a container...
Two variables and dependence	Covariation between variables and dependence of one variable with respect to the other.	Verbalization of a relationship between the variables and their dependence. Cause-effect: if this happens, then the other happens.
Two variables and dependence	Covariation between variables and dependence of one variable with respect to the other.	Verbalization of a relationship between the variables and their dependence. Cause-effect. Construction of different representations (numerical and graphical).
Two variables and functional relationship	Covariation between variables and functional relationship between variables.	Verbalization of a relationship between the variables and their dependence. Cause-effect. Construction of different representations (numerical, graphical, and algebraic).
Two variables functional relationship	Covariation between variables and functional relationship between variables (global coherence between different representations).	Verbalization of a relationship between the variables and their dependence. Cause-effect. Construction of different representations (numerical, graphical, and algebraic) and review of global coherence.

6.3. The Co-construction of Habitus in the Classroom

The results of the investigation highlighted the process of co-construction of knowledge from a sociocultural perspective. The process followed incorporated several elements, among which the following stand out:

- The selection of a teaching method that fosters student interaction for learning (ACODESA).
- The design of interconnected situations that encourage divergent thinking, the use of various representations, and the possibility to converge towards a goal through a communication process.
- The selection of theoretical elements is critical for understanding how teams construct knowledge.

The results illustrate the complexity of the mathematics learning process within a sociocultural context. As in previous studies (Hitt et al., 2023), we argue that the discussion on essential emerging concepts related to functional-spontaneous representations, socially constructed representations, and *habitus* in the mathematics classroom contributes to a deeper understanding of this process. Furthermore, these discussions open the door to more questions that will guide future research in this area.

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Fernando Hitt Espinoza

Cinvestav-UQAM (México)
hitt.fernando@uqam.ca | https://orcid.org/0000-0002-9106-8806
Contribución: conceptualización, investigación, metodología, curación de datos, validación, visualización, redacción.

Samantha Quiroz Rivera

Universidad Autónoma de Coahuila (México)
samantha.quiroz@uadec.edu.mx | http://orcid.org/0000-0002-1332-8000
Contribución: conceptualización, curación de datos, validación, análisis formal, visualización, redacción.

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Evolución de Representaciones No Institucionales en un Proceso de Modelación en la Enseñanza (ACODESA)

Fernando Hitt Espinoza @ [1], Samantha Quiroz Rivera @ [2]

[1] Cinvestav-UQAM (México)
[2] Universidad Autónoma de Coahuila (México)

Durante el último siglo, las perspectivas constructivistas han desempeñado un papel central en la investigación en educación matemática. No obstante, el reconocimiento del aula como una microsociedad introdujo nuevos desafíos, lo que dio lugar al desarrollo de enfoques socioculturales que destacaron la importancia de la interacción social en la co-construcción del conocimiento matemático. Desde esta perspectiva teórica, el presente estudio examina el proceso de construcción de la noción de covariación dentro de un marco sociocultural. Para abordar este objetivo, se empleó el método ACODESA con el propósito de fomentar procesos de pensamiento divergente. Este enfoque de enseñanza y aprendizaje integra cinco etapas: trabajo individual, trabajo en equipo, debate científico en plenaria, autorreflexión e institucionalización. Estas etapas brindan oportunidades para la emergencia y el refinamiento de representaciones funcionales-espontáneas (F-SR), que evolucionan hacia representaciones socialmente construidas (SCR) mediante la interacción, la comunicación y la reflexión. En el estudio se diseñaron e implementaron cinco situaciones de investigación (RS) con cuatro estudiantes de secundaria en Montreal, Canadá (de 14 a 15 años). Cada RS se planteó en contextos cotidianos familiares para los estudiantes, a partir de preguntas abiertas destinadas a estimular la creatividad y promover el uso de múltiples representaciones. La progresión de las RS favoreció que los estudiantes construyeran representaciones verbales, gráficas y algebraicas. Los resultados muestran cómo las F-SR generadas inicialmente por los estudiantes de manera individual se transformaron en SCR como resultado de las discusiones colaborativas y del proceso ACODESA. Esta evolución permitió identificar distintos niveles cognitivos en la construcción del concepto de función por parte de los estudiantes, desde verbalizaciones iniciales hasta representaciones numéricas, gráficas y algebraicas cada vez más complejas. Además, los resultados resaltan el papel del habitus en la configuración de las prácticas en el aula, evidenciando que el conocimiento matemático no solo se internaliza de manera individual, sino que también se negocia y reconstruye socialmente. En conclusión, este estudio aporta evidencia de que las representaciones socialmente construidas en el aula de matemáticas emergen y se transforman a través de procesos de comunicación y objetivación. Estos hallazgos contribuyen a una comprensión más profunda de la complejidad del aprendizaje matemático en contextos socioculturales y señalan nuevas líneas de investigación sobre la interacción entre representación, colaboración y desarrollo conceptual.