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Mathematical Problem-Posing Research: Revisiting Some Answered and Unanswered Questions

Investigación sobre la formulación de problemas matemáticos: revisión de algunas preguntas respondidas y no respondidas

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Abstract ∞ In 2015, Cai et al. reviewed the state of knowledge in problem-posing research. The purpose of the current review is to explore what we have learned as a field over the last 10 years by focusing on three questions from Cai et al.'s (2015) review: Can students and teachers be effectively trained to pose high-quality problems? What do we know about the cognitive processes of problem posing? How are problem-posing activities included in mathematics curricula? Specifically, we review what we know currently with respect to the three questions, including the progress made so far and additional findings on mathematical problem-posing research. We conclude this review by offering additional questions and suggestions for future research.

Keywords ∞ Problem posing; Mathematics; Cognitive processes; Student learning; Curriculum

Resumen ∞ El propósito de esta revisión es sintetizar los avances de la investigación sobre formulación de problemas, retomando las tres preguntas del trabajo de Cai et al. (2015): ¿Pueden los estudiantes y los docentes ser entrenados de manera efectiva para plantear problemas de alta calidad? ¿Qué sabemos sobre los procesos cognitivos de la formulación de problemas? ¿Cómo se incluyen las actividades de formulación de problemas en los planes de estudio de matemáticas? En concreto, revisamos lo que sabemos actualmente con respecto a estas tres preguntas, incluidos los avances logrados hasta ahora y hallazgos adicionales en la investigación sobre la formulación de problemas matemáticos. Concluimos esta revisión ofreciendo preguntas adicionales y sugerencias para futuras investigaciones.

Palabras clave ∞ Formulación de problemas; Matemáticas; Procesos cognitivos; Aprendizaje del alumnado; Currículo

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1. INTRODUCTION

Although problem posing has long been recognized as a significant intellectual activity (e.g., Einstein & Infeld, 1938), it has become increasingly prominent in recent years as a way to understand students' and teachers' mathematical thinking and provide adequate support for improving their thinking (e.g., Cai et al., 2020). This prominence has been reflected in the proliferation of problem-posing journal special issues (e.g., Cai, 2025b), books (e.g., Singer et al., 2015), meta-analyses (e.g., Ran et al., in press), and review papers (e.g., Baumanns & Rott, 2022a). In response to statements such as "the field of problem posing is still very diverse and lacks definition and structure" (Singer et al., 2013, p. 4) and suggestions for making sharper distinctions among various contextualized problem-posing situations (Silver, 2013), Liljedahl and Cai (2021) and Cai et al. (2023) suggested three conceptualizations of problem posing in research—problem posing *as a cognitive activity*, *as a learning goal*, and *as an instructional approach*. The first includes one's interpretation of a given situation and then the generation of a question based on the situation; the second regards problem posing as a capacity that students and teachers develop over time. The third considers problem posing as a tool to achieve cognitive and affective learning goals through engaging in problem-posing activities. Thus, the field has made progress toward more precise characterizations of mathematical problem posing (Silver, 2013). In this paper, we broadly review advances over the past decade produced by the field's sharpening gaze on problem posing.

In 2015, Cai et al. reviewed the state of knowledge in mathematical problem-posing research with respect to 10 questions. For each question, Cai et al. (2015) discussed what had been answered as well as related unanswered questions for future research. Recently, Ölmez et al. (in press) provided an updated review of research addressing several of those questions. The purpose of the present review is to examine what progress has been made in the ensuing years on three key questions from Cai et al. (2015): Can students and teachers be effectively trained to pose high-quality problems? What do we know about the cognitive processes of problem posing? How are problem-posing activities included in mathematics curricula? These questions each align with one of the three conceptualizations of problem posing noted above. The first treats problem posing as a learning goal for both students and teachers, the second explores problem posing as a cognitive activity, and the third focuses on problem posing as an instructional approach (particularly the curriculum aspect, given that teachers' access to problem-posing tasks and activities is crucial).

We briefly note that we begin from the premise that problem posing is important in school mathematics because it fosters students' conceptual understanding, creativity, interest in mathematics, positive attitudes toward mathematics, and problem-solving skills (e.g., Cai et al., 2015; H. Zhang & Cai, 2021) and can be used as an instrument to assess students' learning of particular content (e.g., Cai et al., 2020). Moreover, both students and teachers are capable of posing important and interesting mathematical problems (e.g., L. Zhang et al., 2022). Research has found that even students with no prior experience in problem posing are able to pose viable, innovative, and multi-step mathematical problems based on a variety of situations (Cai et al., 2015; Singer et al., 2015). This makes problem posing a high-ceiling and low-floor activity that can provide all students opportunities for making sense of mathematics (Cai & Hwang, 2021; Singer et al., 2015). One unanswered question Cai et al. (2015) raised with respect to students' capability to pose problems was, "Why do students pose non-mathematical, trivial, or otherwise suboptimal problems or statements?" (p. 5). That is, why do students sometimes produce responses to problem-posing tasks that are not the expected solvable mathematical problems? Ran et al.'s (2025) investigation of students' unexpected responses provides, to some extent, answers to this question. As the first systematic study that examined middle school students' unexpected responses, Ran et al. revealed that three problem-posing processes—orientation, connection, and generation responses—might lead students to provide responses that are not solvable mathematical problems.

In discussing each of the three questions posed in Cai et al. (2015), we first provide an overview of problem-posing research in the respective area over the past decade or so. This overview includes a discussion of the extent to which the related unanswered questions raised by Cai et al. (2015) have been addressed. We then identify some additional questions that remain unresolved and may serve as directions for future research.

2. REVIEW OF THE CURRENT STATE OF KNOWLEDGE

2.1. Can students and teachers be effectively trained to pose high-quality problems?

Although Cai et al. (2015) treated this question as its own issue, we believe that it is best addressed as part of the broader question, "What do we know about the impact of problem posing on students and teachers?" that we address in this review. Although students and teachers are capable of posing important and interesting mathematical problems, they can sometimes have difficulty posing such problems. Regarding students' and teachers' problem posing, Cai et al. (2015) urged further research around the following unanswered question: "What strategies and ways of thinking are most productive for posing problems, and under what types of mathematical situations are different strategies effective?" (p. 7). Although it seems likely that students and teachers can be effectively trained to pose high-quality problems, research is still ongoing in understanding how best to achieve this goal. Progress has been made in the past decade on systematically reviewing through meta-analyses which problem-posing intervention designs are effective, for what outcome measures they are effective, and for what populations they are effective.

To our knowledge, six meta-analyses (Kul & Çelik, 2020; Ran et al., in press; Rosli et al., 2014; Wang et al., 2022; C. Zhang et al., 2024; L. Zhang et al., 2024) have examined the effects of problem-posing interventions on various outcomes, including learners' problem-posing ability, producing mixed results. Three meta-analyses (Kul & Çelik, 2020; Rosli et al., 2014; Wang et al., 2022) were either based on limited numbers of studies or consisted of studies conducted mostly in one country or with a limited number of components related to problem-posing interventions. In L. Zhang et al.'s (2024) broader meta-analysis of 26 studies published between 1996 and 2021 including studies from more varied countries and with more intervention components, the interventions were found to be effective in terms of learners' *mathematical problem-posing competence*. Moreover, the meta-analysis conducted by Ran et al. (in press) reviewed 26 empirical studies with 59 effect sizes, including K-12 students and preservice teachers, finding that problem-posing interventions were also effective towards learners' (students and preservice teachers) cognitive mathematical learning outcomes, including problem-posing and problem-solving performance. Specifically, the interventions were more effective if the learning environments had technological support for problem posing and if learners were provided problem examples when working on problem-posing tasks. Thus, Ran et al.'s (in press) meta-analysis revealed that students can be effectively trained when technological support is present and problem examples are provided. Although small but targeted problem-posing interventions might not lead to increases in students' mathematics achievement, such interventions can be sufficient to make positive changes in students' problem-posing and problem-solving abilities. Similarly, C. Zhang et al.'s (2024) three-level meta-analysis of 32 studies published between 2000 and 2023 found that problem-posing interventions had moderately positive and small positive effects on students' cognitive and non-cognitive levels of student learning outcomes.

The components of the meta-analyses on problem-posing interventions mainly included sample level, group size in which problem posing occurred, types of problem-posing tasks, the amount of information in those tasks, the types of follow-up instruction after problem posing, and duration of the interventions. Table 1 presents the key findings for each component.

Table 1. Summary of the Meta-Analyses on Problem-Posing Interventions

Components	Kul & Çelik (2020)	Ran et al. (in press)	Rosli et al. (2014)	Wang et al. (2022)	C. Zhang et al. (2024)	L. Zhang et al. (2024)
Sample level	No significant difference exists between students and preservice teachers on problem posing.	Interventions were not significantly different between K-12 and university students.	Preservice teachers were more engaged in problem-posing activities than Grades 4-12 students.	Sample level was not a moderator for the effect of problem-posing strategies on mathematics learning.	No significant difference exists between primary school and high school and university students.	Interventions for K-12 students were more effective than those for teachers.
Group size	N/A	No differential effect of interventions was found, including whether students performed the tasks individually or in groups.	N/A	Students in groups had more improvement on problem posing than those who worked on the tasks individually.	N/A	Interventions conducted with small groups (<25) were more effective than those with large groups (>50).
Types of problem-posing tasks	N/A	Posing problems based on problem examples was more effective than the other three types on learners' cognitive learning outcomes.	N/A	N/A	N/A	N/A

Components	Kul & Çelik (2020)	Ran et al. (in press)	Rosli et al. (2014)	Wang et al. (2022)	C. Zhang et al. (2024)	L. Zhang et al. (2024)
The amount of information	N/A	N/A	Free task-based interventions were more effective than those including semi-structured and structured tasks.	Interventions were more effective when structured, semi-structured, and free tasks were implemented.	Structured tasks with specific conditions and information were more effective than free tasks.	N/A
The types of follow-up instruction	N/A	Interventions were effective on learners' cognitive mathematical outcomes if teachers provided feedback or if teachers provided feedback and peer interaction took place.	N/A	N/A	N/A	N/A
Duration	Duration of the problem-posing activity was not effective for increasing students' problem-posing ability.	N/A	N/A	Longer duration interventions tended to improve students' dispositions towards mathematics.	Intervention duration of more than 10 weeks was significantly less effective than less than 5 weeks.	Intervention duration was not a moderator for the effects of interventions on problem posing.

In terms of sample level, Wang et al. (2022) found that there was not enough evidence to consider sample level as a moderator for the effect of problem-posing strategies on mathematics learning. Similarly, Ran et al. (in press) documented that problem-posing interventions were not significantly different across sample levels, indicating that K-12 and university students were equally effectively trained to pose high-quality problems. C. Zhang et al. (2024) documented no significant difference between primary school and high school students or between primary school and university students in terms of problem-posing instructional interventions. In contrast, Rosli et al. (2014) reported preservice teachers' greater engagement in problem-posing activities compared to Grades 4-12 students. Yet, L. Zhang et al. (2024) found that interventions designed for K-12 students were more effective at improving their problem-posing competence than those for preservice and in-service teachers. Thus, K-12 students may have an advantage over preservice and in-service teachers in posing high-quality problems. This may be because older people are more accustomed to conventional methods of teaching, less likely to change the ways in which they have learned mathematics thus far, and more likely to experience low motivation in posing problems (Silver, 1994).

In terms of group size, Ran et al. (in press) found no differential effect of problem-posing interventions, including whether students performed the tasks individually, in pairs, or in groups of three or more students working together. However, Wang et al. (2022) reported that students who performed the tasks in groups had greater problem-posing improvement than those who worked on the tasks individually. However, L. Zhang et al. (2024) concluded that intervention studies conducted with small groups of students (less than 25) were more effective than those implemented with large groups of students (more than 50). This indicates that small groups of students can be more effectively trained to pose high-quality problems than large groups of students. It might be easier for teachers to track students' problem-posing ability and focus on particular weaknesses of each student to improve their posing ability in smaller groups compared to larger groups.

In terms of the types of problem-posing tasks (i.e., posing problems based on problem examples, posing problems by arranging sentence cards, posing problems based on given problem situations, and posing problems based on both problem examples and given problem situations), posing problems based on problem examples was more effective for learners' cognitive mathematical learning outcomes (Ran et al., in press). Seeing one possible problem-posing example might enable learners to understand what is expected from them so that they can more easily come up with similar problems. Ran et al. argued that providing problem examples might be more relevant to posing high-quality problems than to other cognitive outcomes such as conceptual understanding. Thus, providing problem examples is a promising way to improve students' and teachers' problem-posing ability such that they understand the expectation to pose high-quality problems.

In terms of the amount of information that should be part of problem-posing tasks, providing students structured tasks—with specific conditions and information rather than free tasks—allows them to set up more connections between the mathematical relationships in the task and pose more complex problems that require more steps for a solution and a deeper level of mathematical thinking (e.g., Cai et al., 2023; C. Zhang et al., 2024). As an example, in a recent study with 669 Chinese elementary school students, L. Zhang et al. (2022) found that students performed better at posing problems when specific information was present in the task (i.e., specific numerical information). The specific numerical information creates readily accessible opportunities to set up mathematical relationships. However, there is evidence supporting the usefulness of a variety of task types. Wang et al.'s (2022) meta-analysis reported that problem-posing interventions were more effective when all types of tasks (structured, semi-structured, and free) were implemented. Free task-based problem-posing interventions might also be more effective at improving problem posing and mathematical dispositions than those interventions that include only semi-structured and structured tasks (Rosli et al., 2014).

In terms of the types of follow-up instruction, the problem-posing interventions were effective at improving learners' cognitive mathematical outcomes, including problem-posing ability, if teachers provided feedback about the quality of posed problems or if teachers provided feedback about the posed problems and peer interaction took place, such as exchanging thoughts about the posed problems and reviewing other students' problems (Ran et al., in press). Given that there is no one correct answer in a problem-posing task and evaluating posed problems is not a quick process for teachers and peers, follow-up instruction through teacher feedback and peer interaction is important. Such feedback can include whether posed problems are solvable, leading to revising unsolvable problems, and presenting some problems to the whole class (Liljedahl & Cai, 2021). Future studies are needed to understand more about which types of follow-up instruction are most effective and how to design follow-up instruction that improves learners' ability to pose high-quality problems.

Regarding the effect of intervention duration for improving the ability to pose high-quality problems, our knowledge remains limited (L. Zhang et al., 2024). Although long durations might assist learners in gaining additional practice with problem posing and receiving feedback from peers or teachers, this might also decrease motivation for posing problems after a while. In C. Zhang et al.'s (2024) meta-analysis, studies greater than 10 weeks were found to be significantly less effective than studies that lasted less than 5 weeks. And, in Kul and Çelik's (2020) meta-analysis, extended duration of the problem-posing activity in mathematics lessons was not effective for increasing students' problem-posing ability. However, Wang et al.'s (2022) meta-analysis documented that longer duration interventions tended to improve students' dispositions towards mathematics. This suggests some unanswered questions: What lengths of time for problem-posing interventions are ideal for effectively improving learners' posing of high-quality problems during problem-posing activities? Would it be more effective to implement problem-posing interventions once with heavy treatment or spread it out over time with equal distribution of treatment? Future intervention studies need to be clearly designed to consider aspects such as implementation and duration of interventions with particular group sizes in which problem posing occurs, specific problem-posing tasks and activities that will be implemented, and problem situations and prompts that will accompany those tasks (Cai et al., 2023; Ran et al., in press).

The number of problem-posing intervention studies has increased, but their intervention designs and implementations have varied considerably (Cai et al., 2020), making it difficult to reach a consensus. Further studies are needed to better understand the effectiveness of problem-posing interventions under specific conditions based on several outcomes, including both cognitive and affective aspects. Such interventions should be carefully designed by specifying the number and characteristics of problem-posing tasks and the conditions in which those tasks will be implemented, as well as follow-up instructional support after problems are posed (Ran et al., in press). Further studies that use clinical interviews are also needed to learn more about the impact of task variables, including different types of problem situations and prompts on posing high-quality problems such that it will be possible to design problem-posing tasks accordingly for use in classrooms and evaluate the mathematical understandings of students (Cai et al., 2024).

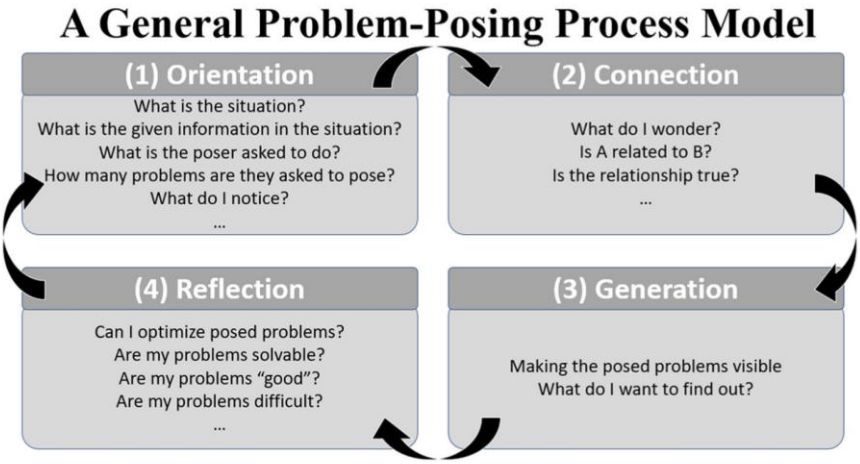
2.2. What do we know about the cognitive processes of problem posing?

Another area that Cai et al. (2015) emphasized for future research concerns our understanding of the cognitive processes involved in problem posing. This has received less attention than the products of problem posing. In 2015, Cai et al. posed two key unanswered questions regarding 1) the cognitive processes of problem posing and 2) how the understanding of such information would enhance teaching and improve student learning. In this section, we present four distinct models developed since 2015 of the (meta)cognitive processes involved in problem posing.

We conclude the section by discussing three additional papers that we believe illustrate how teachers' deepened understanding of students' cognitive processes during problem posing can inform the design of problem-posing-based learning experiences and support responsive instructional decisions that can ultimately enhance student learning. Although the work reviewed in this section is not an exhaustive review of all existing literature, we selected them to provide illustrative insights into current understandings of students' cognitive processes in problem posing and suggest promising directions for future research and practice aimed at leveraging these understandings to improve mathematics teaching and learning.

First, Cai et al.'s (2015) fourth "unanswered question" (although not explicitly phrased as a question in the original) involved the details of the cognitive processes underlying problem posing: How do students and teachers go about posing mathematical problems in any given situation? (p. 9) Cai et al. introduced several models of problem-posing process, such as Cifarelli and Cai's (2005) recursive model of problem posing and solving and Pittalis et al.'s (2004) cognitive process model, which includes problem posing. Since then, significant progress has been made in understanding the cognitive processes of problem posing. Researchers have proposed new problem-posing process models (e.g., Koichu & Kontorovich, 2013). As research on problem posing has expanded over the last decade, more recent models, often based on reviews of empirical studies, have provided a clearer picture of the process. For example, L. Zhang et al. (2022) proposed a three-phase model of problem posing. *Understanding the task* includes understanding the context of the given problem-posing task. The second phase, *constructing the problem*, includes selecting the elements of the given task and recognizing the relationships between the selected elements to (re)formulate mathematical problems. The third phase, *expressing the problem*, includes organizing language to express the problems. In 2024, Cai and Rott also proposed a general model of problem-posing processes based on their review of 75 empirical studies on problem-posing processes. Cai and Rott's (2024) model consists of four phases (see Figure 1). *Orientation* refers to the poser's understanding of a given problem-solving or problem-posing task, and *connection* refers to making connections to their prior knowledge related to the given task and to developing problem creation ideas. *Generation* refers to making the posed problems "visible," which is akin to L. Zhang et al.'s (2022) third phase, *expressing the problem*. The last phase is *reflection*, the metacognitive activity of monitoring what was done during the problem-posing process.

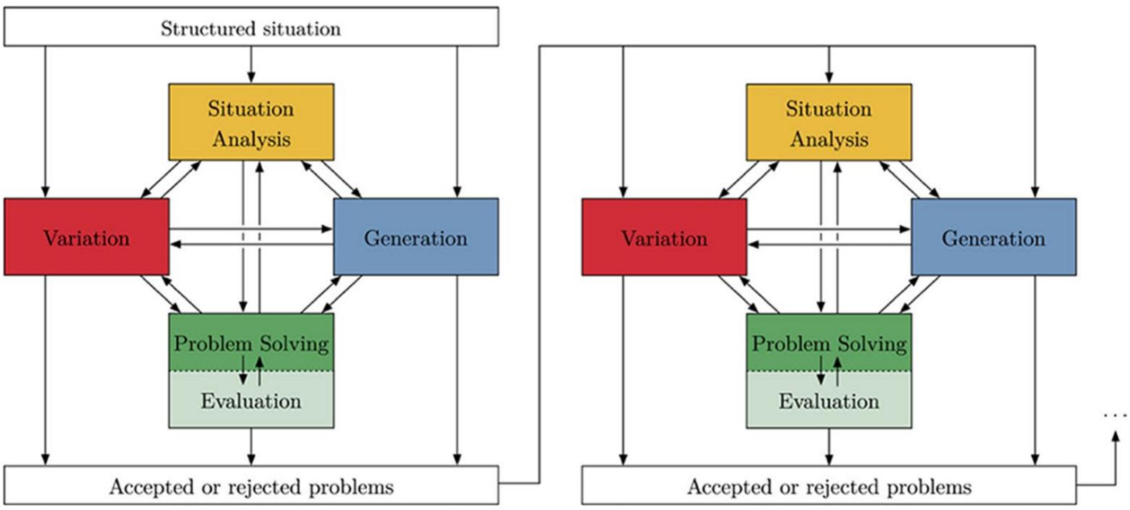
Figure 1. General Problem-Posing Process Model



Source: Cai & Rott, 2024, p. 68

Baumanns and Rott (2022b) proposed a five-phase model of the problem-posing process around structured problem situations (see Figure 2). The first phase, *situation analysis*, involves capturing and understanding the conditions of the given task. The second phase, *variation*, involves changing or omitting one or more conditions of the given task or of a task posed from a previous problem-posing process. The third phase, *generation*, involves adding new conditions to the given task or a previously posed task. The fourth phase, *problem solving*, involves solving the newly posed task. The fifth phase, *evaluation*, involves assessing the posed tasks based on the individual's own criteria, such as solvability, topical relevance to the initial task, or appropriateness for the intended audience.

Figure 2. Five-Phase Problem-Posing Process Model



Source: Baumanns & Rott, 2022b, p. 264

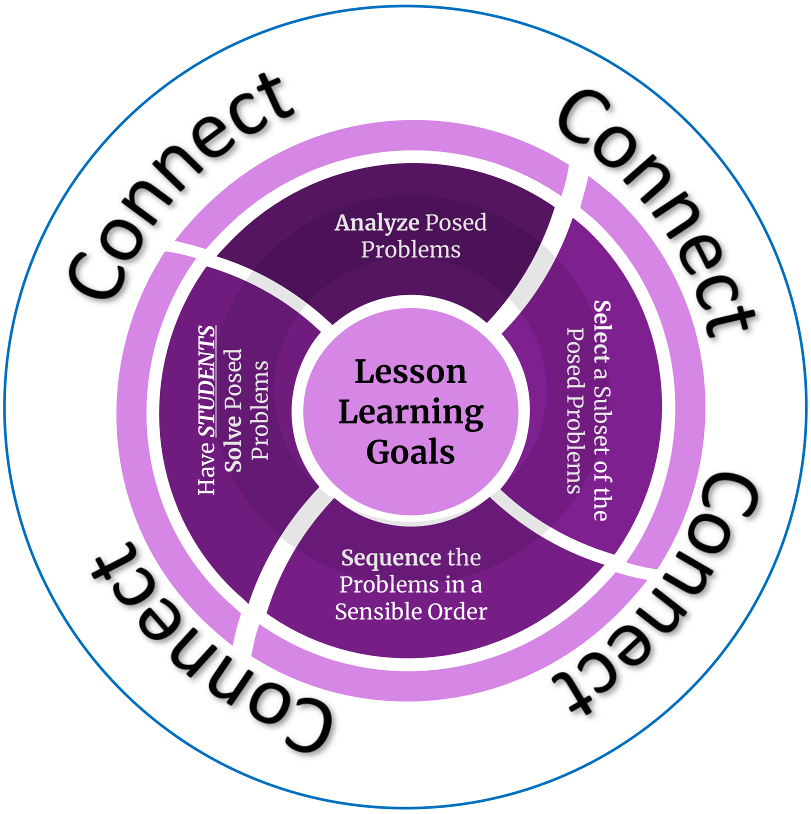
In addition, Baumanns and Rott (2023) proposed a framework for identifying the metacognitive behaviors of problem posers during posing. They categorized these behaviors largely into three types:

- Planning
  - Focus on a starting point of the problem-posing situation to generate new problems;
  - Capturing the conditions and identifying the restrictions of the given problem-posing situation;
  - Reflect necessary knowledge;
  - Express general procedure for problem posing.
- Monitoring & control
  - Controlling the general procedure for problem posing;
  - Controlling the notation or representation of the posed problems;
  - Assessing consequences for the problem's structure through the modified or new constructed conditions;
  - Controlling mathematical activities related to a posed problem.
- Evaluating
  - Assessing and reflecting on the characteristics of the posed problems [if it is appropriate for a specific target group, solvable, interesting, well-defined];
  - Reflect on possible modifications of the posed problems (Baumanns & Rott, 2023, pp. 1392-1395).

Thus, the field now possesses a better understanding of the processes that a poser would likely engage in when formulating new problems or reformulating given problems. This includes both general models of problem-posing processes (e.g., Cai & Rott, 2024; L. Zhang et al., 2022) and models that focus specifically on problem posing with structured problem-posing tasks (e.g., Baumanns & Rott, 2022b). Additionally, we have a framework for identifying different types of metacognitive behaviors exhibited by problem posers (e.g., Baumanns & Rott, 2023).

Cai et al.'s (2015) fifth previously unanswered question was as follows: "How can an understanding of students' problem-posing cognition help teachers to improve student learning?" (p. 9). Compared to the perspective of understanding students' problem-posing processes, research on the relationship between students' problem-posing cognition and teachers' instructional decisions remains relatively uncharted. Few studies have examined how teachers make instructional decisions based on their understanding of students' problem-posing cognition. Similarly, few studies have focused on supporting teachers to use students' posed problems to inform instruction. For example, Hwang et al. (2025) proposed the Problem-Posing-Based Learning (P-PBL) Analyze, Select, Sequence, Solve, Connect (ASSSC) discussion protocol (see Figure 3) to help teachers and teacher educators facilitate mathematical discussions using students' posed problems. This protocol builds on Cai (2022) and adapts the Five Practices Framework (Smith & Stein, 2011), which guides the orchestration of mathematically productive discussions in mathematics classrooms. By centering their instructional goals, teachers analyze, select, sequence, and solve a subset of students' posed problems, helping them organize students' activities around problem-posing tasks based on the students' contributions.

Figure 3. P-PBL ASSSC Discussion Protocol



Source: Hwang et al., 2025, p. 996

Another line of research closely related to this question involves teachers' decision-making during the planning and implementation of P-PBL. For example, Han et al. (2024) explored the views of 15 middle school mathematics teachers who were actively learning about P-PBL. They found that teachers aimed to balance accessibility and challenge when they envisioned how their students would respond to given problem-posing tasks. Instructional decisions were based on teachers' perceptions of students' prior knowledge, anticipated challenge, and the characteristics of the problem-posing tasks themselves, such as problem contexts, mathematical content, visual representation, or the degree of structure provided by both the problem-posing problem situation and prompts. The authors found that the teachers' preferences for particular problem-posing prompts were mediated by (a) the characteristics of their students, (b) the characteristics of the problem-posing tasks—problem situation and prompt—and (c) anticipated student challenges or issues that students would encounter during P-PBL implementation.

Regarding teachers' decision-making during P-PBL implementation, H. Zhang and Cai (2021) documented 22 cases of Chinese teachers implementing P-PBL, providing valuable insights into how teachers and students engage in lessons centered around problem-posing activities. These cases illustrate not only how these teachers planned their lessons—anticipating mathematical problems students might pose—but also how they handled both anticipated and unanticipated problems during P-PBL implementation. Although these teaching cases offer a better understanding of teachers' instructional decision-making based on students' posed problems, they have limitations. Because the cases were submitted as part of teachers' assignments with word limits, some details were limited. Moreover, the field still lacks established instructional strategies for handling students' posed problems, highlighting the urgent need for future research on how teachers can make effective instructional decisions to achieve their intended mathematical goals through P-PBL. There remains a need for more systematic research on how teachers handle students' posed problems and how their understanding of students' mathematical thinking exhibited in the posed problems can support their instructional decision-making and help them achieve their intended pedagogical goals.

2.3. How are problem-posing activities included in mathematics curricula?

In comparing mathematics curricula from before 2015 with those after 2015, our review of how problem posing has been treated across different education systems was through the lens of three levels of curriculum: the planned curriculum, the intended curriculum, and the implemented curriculum. Whereas *the planned curriculum* covers the representation of problem posing in policy documents and official curriculum standards, *the intended curriculum* deals with the inclusion of problem posing in textbooks and other instructional materials used in the classroom, and the *implemented curriculum* refers to the extent to which teachers adapt and teach problem posing inside the classroom (Cai, 2025a). Although there has not been much progress regarding the inclusion of problem-posing activities in curricula, a recent special issue explored inclusion of problem posing in different countries' curriculum standards and curriculum materials including the integration of problem posing into their instruction (Cai, 2025b). Studies in this special issue and past research has revealed that problem-posing activities are rarely included in mathematics curricula despite increasing emphasis on problem posing (Cai, 2025b; Cai et al., 2020).

Regarding *the planned curriculum* based on the integration of problem posing into mathematics curricula in different countries, the National Council of Teachers of Mathematics' (NCTM, 1989) *Curriculum and Evaluation Standards for School Mathematics* was one of the first documents to (briefly) mention problem posing in the United States. For Grades 3, 5, 6, and 7 mathematics of the Common Core State Standards (CCSS) document, currently the most widely accepted U.S. standards with adoption by 45 states, students are expected to "recognize and describe situations," which aligns with problem posing (Ellerton, 2013). Yet, despite repeated emphasis on incorporating problem posing into school mathematics and classroom practice (e.g., [NCTM], 1989, 2020), there exists no curriculum systematically planned around problem posing in the United States. Similar to recommendations in the United States, the new curriculum for 9-year compulsory education in China aims at students' learning of problem posing and implementation of problem posing into their problem-solving skills to become more familiar with mathematical applications and transition students from passive learning styles to active learning styles (Chinese Ministry of Education, 2001, 2022). On the other hand, Toh and Chua (2025) reported in Cai's (2025b) special issue that in Singapore's official curriculum documents there does not exist a specific emphasis on inclusion of problem posing in school mathematics. Similarly, Baumanns and Rott (2025) reported very rare emphasis of problem posing in German educational standards. Lastly, Bokhove (2025) compared the national curricula of England, Singapore, and the United States and reported inconsistent representation of problem posing in those curricula, such as lack of mentioning problem posing explicitly in England. This indicates that the emphasis on problem posing is different in each country's curriculum, ranging from more emphasis (e.g., China, the United States) to less emphasis (e.g., Singapore) and very rare emphasis (e.g., England, Germany). Once again, despite suggestions in the mathematics curricula for teachers to regularly pose problems, no systematic planning is explicitly mentioned for how to incorporate problem posing into mathematics curricula and what specific steps need to be taken by teachers to support students' learning through problem posing.

Regarding *the intended curriculum*, in 2015, Cai et al. asked the following unanswered question: "How do the actual textbooks include problem posing?" (p. 13). In addition to the lack of integration of problem posing into U.S. and Chinese mathematics curricula in consistent and purposeful ways (Jia & Yao, 2021), popular elementary and middle school mathematics textbooks in the United States and China also contain a small proportion of problem-posing activities (about 3%) that are unevenly distributed across grade levels and content areas, with the majority of the content areas being related to number and operations (Cai & Jiang, 2017). In Germany, textbooks rarely contain problem posing, and when they do, it appears in the Numbers and Operations content (Baumanns & Rott, 2025). Similarly, in Singapore, there exist very few problem-posing activities in early primary grades (Toh & Chua, 2025). These results suggest that there has not been much progress regarding the inclusion of problem-posing activities in school textbooks in the past 10 years.

In 2015, Cai et al. also asked the following unanswered question: "If curriculum designers intend to integrate problem posing into textbooks and teaching materials, what are the best ways to do so?" (p. 13). Although some efforts have been made by textbook writers to include problem posing in textbooks, such inclusions are not evenly distributed in terms of content area and grade level. Therefore, the best ways to integrate problem posing into textbooks and teaching materials remain unknown, but we hope that problem-posing activities can be distributed evenly across different mathematics content areas and grade levels and that clear instructions can be provided in those documents for teachers to apply such activities in classrooms. The ideal number of problem-posing tasks in each grade level and in each content area, as well as the number of each type of problem-posing task, are still unknown to the field.

Finally, regarding *the implemented curriculum*, only a few studies have explored how problem posing is incorporated into instruction. In one study, Toh and Chua (2025) reported how Singaporean students posed problems in classrooms mainly as isolated case studies rather than as a systematic examination. In another study, Muirhead et al. (2025) showed the possibility of incorporating problem posing into U.S. middle school classrooms with teacher initiative through case studies from the P-PBL Project. Still, incorporation of problem-posing practices into different contexts is highly needed, and teachers do not appear to have much support to include problem-posing activities in their teaching plans or to have regular opportunities to create problem-posing learning environments in their classrooms.

3. CONCLUDING THOUGHTS

It is clear that, in the decade since the publication of Cai et al.'s (2015) survey of the mathematics education literature on mathematical problem posing, much has been accomplished in this domain. It is sometimes easy to forget that, especially compared to research on mathematical problem solving, our field's systematic attention to problem posing is still quite a recent phenomenon. Nonetheless, together with the review of Ölmez et al. (in press), our overview of the answered and unanswered questions that Cai et al. (2015) posed reveals that significant progress has been made in recent years to build a more solid theoretical and empirical foundation for our understanding of problem posing. This is true for each perspective of problem posing (Cai et al., 2023), whether it is seen as a cognitive endeavor, as a goal towards which students develop skills, or as a tool in educators' toolboxes for helping students learn mathematics content and practices and develop positive mathematical dispositions and identities.

In this review, we particularly focused on three questions. In response to the question "Can students and teachers be effectively trained to pose high-quality problems?" our review revealed that students and teachers can be effectively trained to pose high-quality problems, but, more broadly, there are mixed, nuanced findings in terms of the effectiveness of problem-posing interventions based on particular designs, outcome measures, and populations. In response to the question "What do we know about the cognitive processes of problem posing?" our review showed that we currently have a more fine-grained understanding of learners' process of posing problems in given situations with the help of newly developed process models that focus on general models of problem-posing processes (e.g., Cai & Rott, 2024), problem posing that accompanies structured tasks (e.g., Baumanns & Rott, 2022b), and differentiating metacognitive behaviors of learners who pose problems (e.g., Baumanns & Rott, 2023). However, well-established instructional strategies that teachers can use in their instruction are still needed to support students' problem-posing processes and fulfill mathematical goals.

Lastly, in response to the question "How are problem-posing activities included in mathematics curricula?" our review reported that the inclusion of problem-posing activities is usually rare in mathematics curricula of different countries over the past decade. Whereas there has been an increasing emphasis in the mathematics curricula of some countries such as China and the United States, almost no emphasis has been placed on it in other countries like England and Germany (e.g., Bokhove, 2025). Similar to the rare inclusion of problem-posing activities in curricula, only a small proportion of problem-posing activities are present in textbooks and teaching materials in the past 10 years.

However, just as there has been clear progress in the field, much remains to be done. Among the still-unresolved questions and issues related to problem posing in mathematics education, we particularly highlight the need to explore how problem posing can best be incorporated into mathematics curricula so that teachers have consistent, well-designed supports for implementing problem posing in their teaching. We also call for research to investigate the most effective ways for teachers to handle students' posed problems. Although some progress has recently been made in this space, the development and testing of robust instructional routines would powerfully inform teachers' efforts to leverage their students' mathematical thinking (i.e., their posed problems) to achieve learning goals. Finally, there remains a pressing need for unifying definitions and frameworks that can be adopted across research teams and sites so that a more coherent and comprehensive picture of problem posing can eventually be drawn. For example, a great deal of research has focused on intentional problem posing in formal instructional situations. However, some researchers have also examined problem posing that occurs spontaneously (Koichu, 2020) and problem posing that occurs in informal settings (Wang & Walkington, 2023). Current frameworks for problem posing do not neatly apply to these situations. We hope that our survey of the state of these aspects of problem-posing research in mathematics education will provide a useful jumping-off point for future research that continues to address the unanswered questions about problem posing.

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Investigación sobre la formulación de problemas matemáticos: revisión de algunas preguntas respondidas y no respondidas

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El propósito de esta revisión es explorar lo que hemos aprendido como campo de investigación en los últimos 10 años, centrándonos en las preguntas de Cai et al. (2015): ¿Pueden estudiantes y docentes ser instruidos de manera efectiva para plantear problemas de alta calidad? ¿Qué sabemos sobre los procesos cognitivos de la formulación de problemas? ¿Cómo se incluyen las actividades de formulación de problemas en los planes de estudio de matemáticas? En respuesta a la primera pregunta, nuestra revisión reveló que estudiantes y profesores pueden ser eficazmente formados para plantear problemas de alta calidad, pero los hallazgos sobre la efectividad de las intervenciones en formulación de problemas son contradictorios, dependiendo de los diseños, las medidas de resultado y las poblaciones implicadas. La variedad de diseños de intervención e implementaciones en estudios de formulación de problemas (Cai et al., 2020) dificulta alcanzar un consenso. Dichas intervenciones deben diseñarse cuidadosamente, especificando la cantidad y características de las tareas de formulación de problemas, las condiciones de implementación y el apoyo didáctico posterior a la formulación de los problemas (Ran et al., en prensa). En respuesta a la segunda pregunta, nuestra revisión mostró que actualmente contamos con una comprensión más detallada del proceso de los estudiantes al formular problemas en una situación determinada, gracias a modelos recientes que se centran en los procesos de formulación de problemas (Cai y Rott, 2024), en la formulación de problemas que acompaña a tareas estructuradas (Baumanns y Rott, 2022b) y en la diferenciación de los comportamientos metacognitivos de los estudiantes que plantean problemas (por ejemplo, Baumanns y Rott, 2023). Sin embargo, se necesita una investigación más sistemática sobre cómo los docentes atienden a los problemas planteados por los estudiantes e interpretan el pensamiento matemático de los estudiantes a través de su formulación, para tomar decisiones instruccionales y ayudarles a alcanzar los objetivos de enseñanza previstos. En cuanto a la tercera pregunta, la inclusión de la formulación de problemas en los planes de estudio sigue siendo poco frecuente en la mayoría de los países. Algunos, como China y Estados Unidos, han aumentado su presencia, mientras que en otros, como Inglaterra y Alemania, apenas se menciona (Bokhove, 2025). Esta escasa presencia también se observa en los libros de texto y los materiales didácticos, donde aún no se ha establecido la mejor manera de integrar estas actividades. Se sugiere que se distribuyan de manera uniforme entre las áreas de contenido y los niveles educativos, con instrucciones claras para su uso en el aula. En conclusión, se han logrado avances significativos en el fortalecimiento de los fundamentos teóricos y empíricos de la formulación de problemas matemáticos. Sin embargo, sigue existiendo una necesidad urgente de unificar las definiciones y los marcos conceptuales para orientar la investigación y la práctica educativa en este ámbito.