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Bridging Problem-Solving and Problem-Posing: Third Graders Learning to Formulate Mathematical Problems in Geometry and Measurement

Conectando resolución y formulación de problemas: Estudiantes de tercer grado aprendiendo a plantear problemas en Geometría y Medición

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Abstract ∞ This study examines the relationship between problem solving (PS) and problem posing (PP) in mathematics education, focusing on 3rd-graders' ability to create problems in Geometry and Measurement. A theoretical framework was adopted to assess the evolution of the problems posed, which considers mathematical and linguistic complexity. The study also compared posed and solved problems, for a deeper understanding of the connections between PS and PP skills. Conducted over eight lessons, it combined curriculum-aligned PS and PP tasks. Students' written work was analyzed qualitatively. Findings show substantial improvements in PP skills, including better mathematical language, grammar and expression, alignment to task prompt, and problem solvability. Initially, students replicated structures from solved problems but gradually showed more creativity and autonomy. The study highlights how task design and the interplay between PS and PP enhance mathematical competencies and offer a viable approach to integrating PP as a learning objective in primary mathematics education.

Keywords ∞ Problem-posing; Problem-solving; Geometry and Measurement; Primary Education; Mathematics Teaching

Resumen ∞ Este estudio explora la relación entre resolución de problemas (RP) y formulación de problemas (FP) en educación matemática, con énfasis en la capacidad de estudiantes de tercer grado para crear problemas en Geometría y Medición. Se adoptó un marco teórico que considera la complejidad matemática y lingüística para analizar la evolución de los problemas formulados. También se compararon problemas resueltos y formulados para comprender mejor la conexión entre RP y FP. El estudio, realizado en ocho clases con tareas alineadas al currículo, combinó actividades de RP y FP. Se analizaron cualitativamente los trabajos escritos del alumnado. Los resultados muestran avances significativos en FP: mejor uso del lenguaje matemático, mayor claridad expresiva, adecuación a las consignas y resolubilidad de los problemas. El estudio resalta el valor del diseño de tareas y la articulación RP-FP para potenciar competencias matemáticas, ofreciendo una vía para integrar la FP como objetivo de aprendizaje en primaria.

Palabras clave ∞ Formulación de problemas; Resolución de problemas; Geometría y Medición; Educación Primaria; Enseñanza de Matemáticas

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1. INTRODUCTION

In recent decades, problem posing (PP) has been established as a relevant activity in mathematics learning. Its contribution to understanding concepts, enhancing creativity and flexible thinking, and a predisposition to learn mathematics is widely recognized. Despite the growing interest in the field of PP, many questions remain unanswered, particularly regarding how task format can influence the way students pose problems (Cai & Rott, 2024) and in the context of primary school (Palmér & van Bommel, 2020).

The relationship between PP and problem solving (PS) has also been the subject of numerous studies (Brown & Walter, 2005; Carrillo & Cruz, 2016; Silver, 1995), which highlight the two activities' reciprocal benefits. However, despite the consensus on its relevance to the development of critical and creative mathematical thinking, many questions about the details of this relationship remain (Silver & Cai, 1996), particularly regarding its implementation in primary education. Literature points to gaps in understanding, for example, about how different task formats impact problem-solving skills in young children (Cai & Rott, 2024). Furthermore, considering the growing emphasis on active learning and the development of autonomy in elementary education, exploring the integration of these activities into specific curricular contexts becomes relevant to supporting evidence-based educational practices.

Despite geometry's importance in mathematics education, it remains underexplored in activities involving PP and PS. Recognizing the need to deepen knowledge in this area, this study explores how 3rd-grade students learn to pose mathematical problems within a teaching experiment focused on the articulation between PP and PS in Geometry and Measurement. Particularly, the study aims to understand:

- the characteristics of the problems posed by students; and
- the relationship between the problems students posed and the problems they solved.

2. THEORETICAL FRAMEWORK

2.1. Characteristics of mathematical problems

Mathematical problems can be defined by various characteristics, particularly those that focus on elements that make them fruitful for learning, such as their structure, the context in which they are embedded, and the opportunities they offer for developing diverse solution strategies (Jacinto & Carreira, 2017). A fruitful problem should provide opportunities to mobilize prior knowledge (Posamentier & Krulik, 2009), enabling *connections with other mathematical concepts*.

A problem can also allow for *multiple strategies* and *multiple solutions*. Leavy and Hourigan (2024) explain that problems with multiple strategies or methods allow students to approach resolution in various ways, fostering critical thinking and creativity. Conversely, problems that admit multiple solutions encourage students to explore alternatives and consider different possible outcomes, making the learning process more dynamic and flexible.

Problems can further be categorized as *purely mathematical* or *real-world problems*. The former involves resolutions in an abstract context, using mathematical language, while the latter starts with real-life situations, requires using mathematics, and culminates in an answer that makes sense in a real-world context (Radmehr & Vos, 2020).

2.2. Mathematical problem posing

PP refers to the creation of a problem or the restructuring of existing ones (Silver, 1995). Reversing the process of analyzing given information and instead questioning how one might have arrived at conclusions is a way of engaging in PP (Brown & Walter, 2005). Barabé and Proulx (2015) note that, in the review phase, after solving a problem, students are encouraged to observe their work. Then, they can generate new ideas, investigate possible connections between mathematical problems, and ultimately pose new problems based on the one they have solved. This practice integrates creativity and mathematical exploration into the educational process. Thus, PP is considered not merely a complement to PS but an essential component of critical mathematical thinking.

Brown and Walter (2005) emphasize that PP involves the generation, analysis, and synthesis of information, skills that are crucial for creative and critical mathematical thinking. Furthermore, PP tasks also provide teachers with tools to assess students' prior knowledge and experiences, as the problems posed by students reflect their mathematical experiences, serving as a window to understand their cognitive strategies and as a mirror of their previous learning (Silver & Cai, 1996).

The problems resulting from the PP process can be evaluated in various ways. Silver and Cai's (1996) framework initially categorizes posed problems as *mathematical*, *non-mathematical*, or *merely statements* without a question. They are then assessed for *solvability* and analyzed for *linguistic* and *mathematical complexity*. Özgen et al. (2017) present a more detailed list of criteria for evaluating the problems posed, extending beyond mathematical and linguistic complexity, namely: 1) *Use of mathematical language*, assessing whether the mathematical terms in the problem statement are complete and correct; 2) *Grammar and expression*, analyzing sentence construction and spelling; 3) *Suitability for acquisitions*, determining whether the problem aligns with the learning goals for the specified educational level; 4) *Quantity and quality of data*, evaluating if the data provided is sufficient and appropriate for solving the problem; 5) *Solvability*, verifying whether the problem can be solved; 6) *Originality*, assessing if the problem was created by the student rather than copied from a textbook or other source; 7) *Problem solution*, confirming if the student solved their own problem correctly and appropriately.

2.3. Relationship between problem posing and problem solving

PP and PS are interconnected processes, with PP serving as a valuable tool to assess students' understanding of PS (Carrillo & Cruz, 2016). Brown and Walter (2005) emphasize that both processes require critical thinking, creativity, and the ability to identify patterns and relationships.

Silver and Cai (1996) found that students who were more proficient in PS generated a higher number of more complex problems. They also found that, when students added new questions to existing problems, the additions were closely related to the original question (e.g., requiring the use of data obtained from solving the original question).

Palmér and Bommel (2020) found that when 6-year-old students were asked to pose problems similar to ones they knew, they often based their tasks on the original problem's data or posed similar questions. This suggests that students initially tend to formulate problems by referencing or replicating familiar ones (Papadopoulos & Patsiala, 2020).

In prior studies, task design often involved explicit instructions to students to replicate the structures of previously solved problems (Palmér & van Bommel, 2020). Moreover, while studies have examined the relationship between the structure of problems solved and the problems posed (Papadopoulos & Patsiala, 2023), students were explicitly encouraged to pose multiple problems from a model problem. Thus, it is important to understand PP in settings where less explicit instruction is provided.

It is also worth noting that PP in primary education is rarely treated as an explicit learning objective, despite its potential to develop mathematical and creative skills. This study, aligned with current Portuguese curricular guidelines (Canavarro et al., 2021), offers an innovative contribution by demonstrating how PP can be integrated into a structured curriculum, promoting creativity and autonomy in formulating problems.

The structure of PS and PP tasks plays a crucial role in the development of students' skills. Cai et al. (2013) demonstrated that open-ended tasks, which offer greater creative freedom, encourage exploring multiple solution strategies, fostering deeper mathematical thinking. On the other hand, closed tasks with predefined solutions tend to limit this exploration, restricting opportunities for creativity development and PS abilities. Furthermore, students exposed to a variety of PS tasks, including open and closed ones, performed better both in PS and PP, suggesting that a balance between these types of tasks may be beneficial for mathematical learning.

3. Research methods

3.1. Design of the study

This study stems from a teaching experiment, defined as the process of planning and teaching, coupled with an investigation into students' learning in the classroom context (Gravemeijer & Cobb, 2006), and has adopted a qualitative approach. The research was conducted in the 2023/2024 school year with a 3rd-grade class of 22 students, aged 8, at a Lisbon private school. The students come from predominantly upper-middle socioeconomic backgrounds and globally have good academic performance.

The teaching experiment comprised eight lessons, with tasks aligned with the mathematics curriculum (Canavarro et al., 2021), alternating between PS and PP. Following Silver (1995), this approach assumes that students draw on experiences and elements derived from the PS activity when they pose problems after solving related ones. The class teacher, who has extensive teaching experience in primary school, led the lessons. Tasks were provided in writing, read and explained before students, who, in pairs, completed them. The teacher intervened only to clarify doubts. At the end of each lesson, the pairs' written work was collected.

This study focuses on four lessons from the teaching experiment—the first and last pairs of lessons (Table 1).

Table 1. The teaching experiment

Week	Task	Type	Mathematical topics
1	T1	PS	• Polygons • Perimeter and area of flat figures • Measurement
	T2	PP	
2	T3	PS	• Prisms and pyramids • Relationships between faces, vertices, and edges
	T4	PP	• Regular prisms and pyramids • Relationships between faces, vertices, and edges
3	T5	PS	• Characteristics of prisms • Relationship between volume and the number of slices
	T6	PP	• Relationships between faces, vertices, and edges
4	T7	PS	• Classification of quadrilaterals • Geometric properties • Comparison of perimeters and areas
	T8	PP	• Representation of triangles • Angles • Polygons

The teaching experiment consisted of just one (macro)cycle due to time constraints but comprised micro-cycles where the researchers reviewed the results from one set of tasks to another, adapting them accordingly (Gravemeijer & Cobb, 2006). Thus, with this exploratory study, in the context of the articulation of PS and PP, the data obtained in this cycle may serve as a foundation for future studies.

3.2. Data collection

Within a geometry context, the tasks considered for this study were T1 (Appendix I) and T2 (Appendix II), focused respectively on PS and PP and on rectangles' area and perimeter. Similarly, T7 (Appendix III) and T8 (Appendix IV) also revolved around PS and PP but focused on quadrilaterals and triangles' properties.

T2 asked to formulate a question aligned linguistically and mathematically with a predetermined answer, making it less open-ended than T8. This added difficulty by requiring coherence between the question and the given solution. However, the task's closed nature could also guide students towards a known endpoint, helping them frame their question accordingly.

Conducting T1 and T2 in consecutive lessons revealed how the problems students solved influenced those they later formulated. In T1, students applied concepts of perimeter, area, and relationship between dimensions of geometric shapes, which they then leveraged to create mathematical problems in T2. A similar process occurred with T7 and T8. In T7, students classified quadrilaterals, developed the ability to identify geometric properties, providing a foundation to understand properties of triangles. They applied concepts such as angles, vertices, and sides, drawing on their prior categorization experience to identify key characteristics of triangles when creating problems involving various triangular flags.

3.3. Data analysis

To analyze the problems' characteristics, they were initially categorized based on their solvability. Then, we employed a rubric inspired by Özgen et al. (2017) for its ability to track students' competency progression across both structured and open-ended tasks.

The adaptation of the rubric enabled the evaluation of *mathematical language usage*, *grammar and expression*, *suitability for acquisitions*, the *quantity, and quality of data*, and the *originality* of the posed problem. Additionally, the *alignment with the task prompt* was assessed, as the tasks contained specific constraints that students had to address when creating their problems. The criteria established in Özgen et al.'s (2017) rubric were followed, assigning a specific score to each of the seven parameters in the PP (Table 2). The scores were assigned to students' problems posed, recorded, and summed at the end to evaluate how they evolved. The rubric aligns with the curriculum goals, encompassing both technical aspects (such as grammar or mathematical language) and pedagogical aspects (such as alignment with learning goals).

To address the second research question, the problems posed by the students were compared with the problems they solved in the task completed immediately before, to understand how they related to each other. Categories were established through an inductive analysis to understand whether the posed problems allowed for *multiple strategies* and/or *multiple solutions* (Leavy & Hourigan, 2024), had a *purely mathematical* or *real-world context* (Radmehr & Vos, 2020), or *established a connection with other concepts* (Posamentier & Krulik, 2009). Regarding *multiple* *strategies* and/or *solutions*, it was assessed whether the problems allowed various solving approaches, encouraged or had different solutions, and stimulated creative solutions. Concerning the *context*, it was considered whether the problem was purely theoretical or simulated a real-world situation, represented a logically credible scenario, or involved an everyday situation applicable to students' daily lives. Finally, concerning the *connection with other concepts*, it was evaluated whether the statements required the use of other mathematical concepts unrelated to geometry or other geometry-related concepts beyond the expected ones. For this, the learning objectives outlined in the lesson plans were considered. Thus, if students used aspects that deviated from the expected task completion but still maintained mathematical relevance and alignment with the problem statement, it was considered that connections with other concepts were established.

Table 2. Rubric towards evaluation of PP skills (adapted from Özgen et al., 2017)

		0 points	1 point	2 points	3 points
Statement	Mathematical language usage	Empty	Incorrect use of mathematical language or concepts	Correct but incomplete use of mathematical language or concepts	Complete and correct use of mathematical language or concepts
	Grammar and expression	Empty, or there are expression and spelling errors	No spelling errors, but there is an expression error	No expression errors, but there is a spelling error	No expression or spelling errors
	Suitability for acquisitions	Empty, or the resolution method is unclear	The resolution process is appropriate to the learning objectives but incomplete or incorrect	The resolution process is not appropriate to the learning objectives but is correct and complete	The resolution process is appropriate and complete in relation to the learning objectives
	Quantity and quality of data	Empty, or no usable data	The data is incorrect or incomplete	The data is correct but not very diverse	The data is sufficient, appropriate, and varied
	Originality	Empty	Very common problem (frequently encountered)	Partially original (different from most problems posed by other students)	Highly original (significantly different from problems posed by other students)
	Alignment with the task prompt	Empty, or has no relation to the task prompt	Does not follow the prompt's conditions but shows some reasoning or process related to it	Follows the prompt's conditions mathematically, with some gaps	Fully complies with the conditions required in the prompt

Initially, the first author analyzed all collected material. Then, the other researchers independently reviewed the coding of the solutions of T2 and T8 and, when divergent interpretations occurred, differences were discussed, and adjustments were made to the previous analysis as deemed appropriate.

4. Results

4.1. Overview

The record corresponding to the number of solvable student-posed problems in the two PP tasks (T2 and T8) is shown in Table 3. A summary of the analysis of the 11 pairs' resolutions in those tasks is provided in Table 4, based on the criteria defined for evaluating the posed problems and the scoring parameters (see Table 2). For each criterion, the number of pairs that achieved each score and the corresponding percentage are presented. The last row shows the total score achieved in each of the criteria in the PP tasks. This score is the sum of the scores obtained by the 11 pairs in each criterion, allowing a global view of the evolution of the problems posed, considering each criterion.

Table 3. Record of solvable problems

Task	n	%
2	7	64
8	10	91

Table 4. Scores of student-posed problems by criteria in each task

Criterion	Task	Scores								Sum of scores
		0		1		2		3		
		n	%	n	%	n	%	n	%	
Mathematical language usage	2	0	0	4	36	3	27	4	36	22
	8	0	0	0	0	5	46	6	55	28
Grammar and expression	2	0	0	3	27	1	9	7	65	26
	8	0	0	0	0	3	27	8	73	30
Suitability for acquisitions	2	0	0	2	18	3	27	6	55	26
	8	0	0	0	0	3	27	8	73	30
Quantity and quality of data	2	1	9	3	27	5	46	2	18	19
	8	0	0	1	9	5	46	5	46	26
Originality	2	0	0	2	18	8	73	1	9	21
	8	0	0	3	27	1	9	7	64	26
Alignment with the task prompt	2	0	0	6	55	2	18	3	27	19
	8	1	9	0	0	4	36	6	55	26

Table 4 shows a noticeable improvement in scores across all criteria in PP from T2 and T8. This consistent increase indicates enhanced competence in PP and in structuring mathematically correct statements.

4.2. Characteristics of the problems posed

In this section, we discuss the characteristics of the problems posed by the 11 pairs (identified by letters A to K), based on the analysis criteria (Table 4).

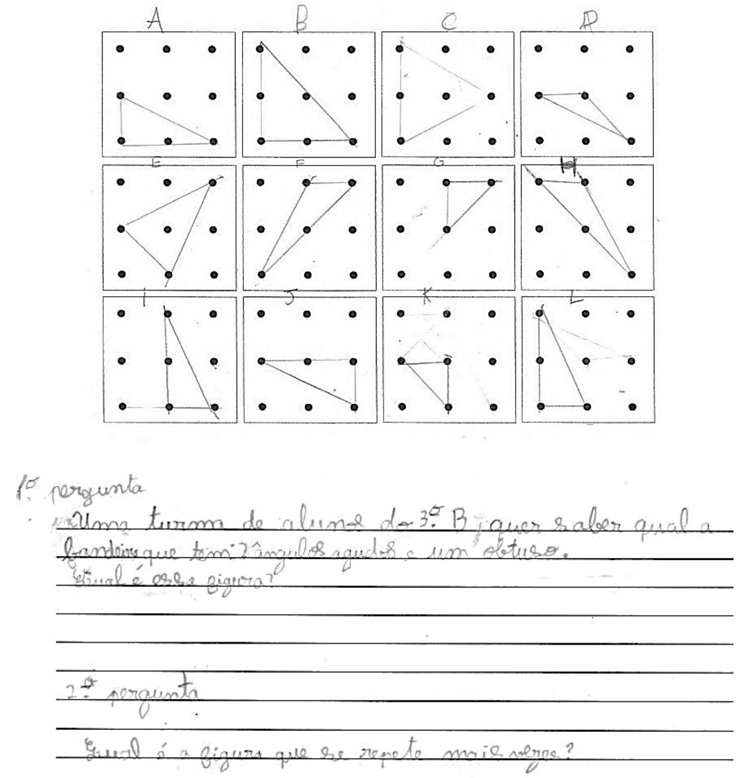
Regarding the *use of mathematical language*, in T2, students rarely achieved the highest scores (2 and 3), whereas in T8, they performed better, with more than half of the responses reaching 3 points. It is worth noting that in T8 there were no scores below 2, while in T2 over 60% of the problems posed received only 1 or 2 points. For example, in T2, the problem posed by pair C, "How much fencing is needed to enclose a square garden that is 40 long?" lacks precision in mathematical language due to the absence of a unit of measurement for the fence length. In contrast, T8 saw better usage of mathematical language and concepts, like pair G's correct use of the geometric concept of equivalent figures: "How many equivalent flags can be found?". However, 5 pairs only scored 2 points, presenting correct but somewhat incomplete concepts.

Regarding *grammar and expression*, high scores were observed in both tasks, with 64% scoring 3 points in T2 and 73% in T8. This suggests that students' spelling and writing at this stage allow for clear and grammatically correct questions. However, in T2, over 25% wrote about problems with expression errors that affected the statements' precision and correctness. For example, pair D's question, *"If a square has a net with 10 meters by 10 meters, how many squares does it have of maximum net?" shows* linguistic flaws, particularly in the reference to "maximum net."

T8 showed better textual coherence, with fewer errors and clearer sentences. This occurred even though the task allowed for more open-ended questions, which could have made written formulation more challenging. Notably, no pair has scores below 2 points. The written production's quality can be exemplified by pair J (Figure 1), who used thorough and correct language.

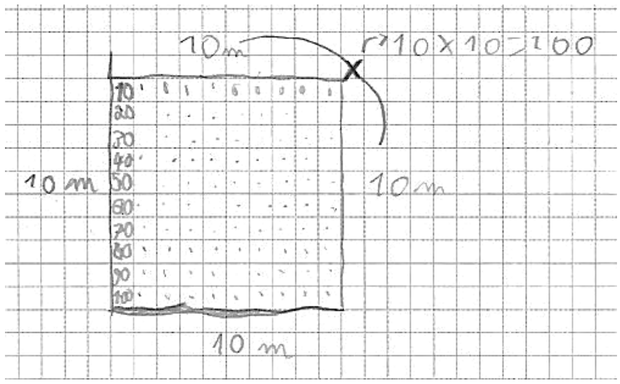
For the criterion of *suitability for acquisitions*, there are some differences between the problems posed, with 55% of pairs achieving 3 points in T2 and 73% in T8. This variation was not significant, as most pairs were already capable of formulating problems aligned with the intended mathematical content in T2. The relatively structured prompt of T2, which included concepts of length and area explored in T1, likely facilitated the inclusion of mathematical topics—a feature maintained in T8. For this criterion, we considered how Geometry and Measurement content from the 3rd grade curriculum was applied. In Figure 2, pair I was calculated the dimensions of a garden with an area of 100 m^2 using the square area formula, demonstrating effective application of this knowledge in their problem formulation.

Figure 1. PP in T8, pair J



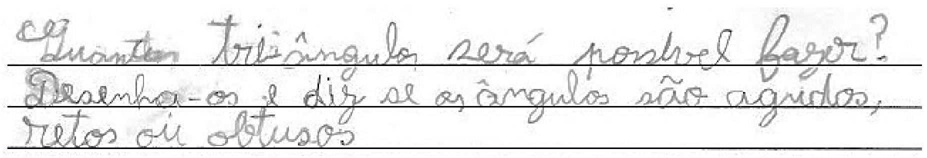
Translation: 1st question: A class of students wants to know which flag has 2 acute angles and one obtuse angle.
What is that figure?
2nd question: Which figure repeats itself the most times?

Figure 2. Resolution of T, pair I



In T8 pair D (Figure 3) successfully posed a problem aligned with previously studied geometric concepts, demonstrating their knowledge and ability to apply concepts related to angle classification.

Figure 3. PP in T8, pair D

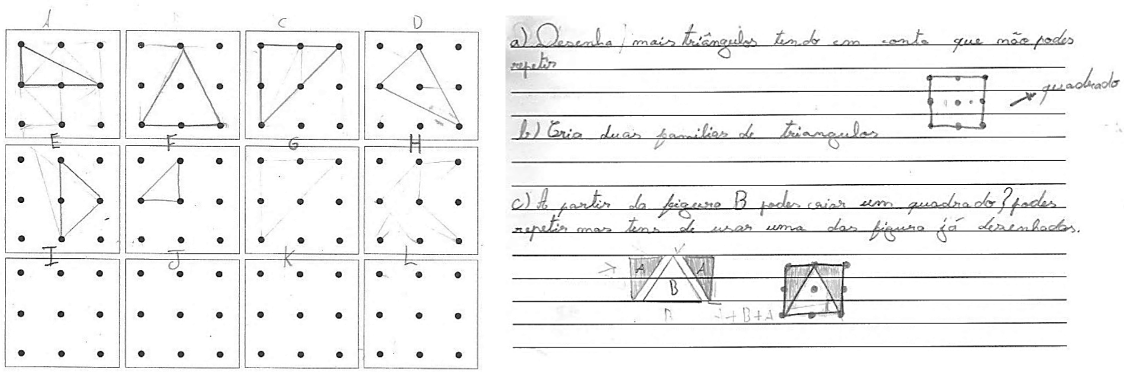


Translation: How many triangles is it possible to make? Draw them and say whether the angles are acute, right, or obtuse

Expressive differences were also observed in the *quantity and quality of data*. While 18% of the pairs achieved 3 points in T2, this percentage increased to 46% in T8. Similarly, in T2, 36% received a score below 2, compared to only 9% in T8. In the interpretation of these results, the differing nature of the two tasks should be considered. T2 was significantly more structured, as it included a previously presented answer for which the corresponding question had to be formulated, which often constrained pairs to the information given in the answer, limiting the variety of data. Thus, although most data provided in T2 statements were adequate, some were deemed insufficiently varied, and only 18,2% scored over 2 points, as students primarily relied on the prompt information without adding their own ideas. Data included was overly general, with some answers that could be restricted to "Yes" or "No."

In contrast, T8 was more open-ended, allowing PP with greater scope and richer data. Figure 4 shows a problem including multiple parts, starting with one direct instruction but adding two questions, introducing original elements in the last one. Students used a triangle labeled B to propose creating a geometric figure that deviated from the task's prompt—a square. This part required PS using the triangles with dotted areas, thereby linking the given element (triangles) with a new one. They also provided a representation to demonstrate the solution process.

Figure 4. PP in T8, pair C



Translation: a) Draw more triangles ensuring that you do not repeat

b) Create two families of triangles

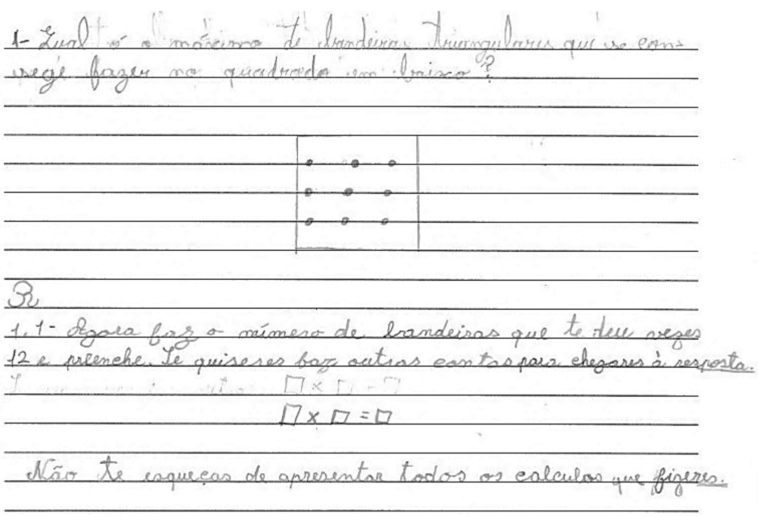
c) From figure B, can you create a square? You may repeat, but you must use one of the already drawn figures

The difference in 3-point scores between T2 and T8 suggests that task type affected the ability to integrate varied, rich information. However, it is worth noting that even in T8, only 46% were rated 3 points. This may be due to the task's limited openness and its placement after a lesson where students solved a problem (T7) very similar to the one they were asked to formulate.

The *originality* of the problems posed varied significantly between the two tasks, with only 9% of T2 problems receiving 3 points, compared to 64% in T8. The lack of originality was a common characteristic across most statements, with pairs struggling to create a question that was both aligned with and distinct from the provided response, as happened with the problem posed by pair H in T2: "With 40 meters of net, what is the maximum area that can be enclosed in a quadrangular garden?".

As T2 and T8 differed significantly, T8's more open-ended design allowed students to explore ideas more freely. However, they often continued to rely on familiar models from both the teaching experiment and prior experiences (Figure 5). Here, they integrated non-geometric knowledge, such as using "text boxes" for calculations and idiomatic expressions commonly found in the wording of mathematical tasks. This approach suggests that students drew on prior exercises beyond the teaching experiment.

Figure 5. PP in T8, pair A



Translation: 1 – What is the maximum number of triangular flags you can make within the square below?

1.1 - Now take the number of flags you calculated, multiply it by 12, and fill in your answer. If you'd like, you can perform other calculations to reach the final result.

Don't forget to show all the calculations you perform.

As such, while some pairs showed progress, 27% still received just 1 point for posing unoriginal problems—either similar to those of classmates or closely resembling previous lesson problems.

In terms of *alignment with the task prompt*, there was an expressive improvement, with 27% of pairs scoring 3 points in T2 and 55% in T8, and the total score increasing from 19 points in T2 to 26 in T8—the largest gain for any criterion. In T2, a major challenge was ensuring statements matched task conditions, particularly in linguistic coherence between the posed question and the provided answer. For example, pair K proposed the problem: “Is it possible to create a quadrangular net with 40 meters and 100 squares?” While correctly including “40 meters” as data, the question was mathematically incorrect and linguistically misaligned with the expected answer about the “length of netting.”

In T8, pairs successfully created problems that aligned with the task’s requirements, both linguistic and mathematical. For example, pair F’s problem: “In the points below, try to create the triangular flags with different shapes that the students have designed” is straightforward and simple, yet precisely aligns with the context in the task prompt, particularly with the excerpt provided in the situation. Scores improved overall, with most pairs reaching 3 points and only one scoring below 2.

Significant changes were observed in the *solvability* of posed problems: 91% were solvable at T8, compared to 64% at T2. Pair B’s proposal, “With that length of netting, is it possible to enclose a net with how much area?”, not only contained grammatical and expression errors but also lacked quantitative data, rendering the problem unsolvable. In contrast, the problems formulated in T8 were solvable, evidenced by pairs G, J, and D, along with pairs C, A, F, I, H, and K, discussed subsequently. These problems reflected an understanding of the necessary conditions to create a viable mathematical problem.

4.3. Comparison of posed and solved problems

The characteristics of the problems solved (T1 and T7) and those formulated in the subsequent PP tasks (T2 and T8) are shown in Table 5. For PP tasks, the number of occurrences for each characteristic has been calculated and the percentage of pairs in which that characteristic was observed. A high percentage of problems posed that replicated the characteristics of problems solved may suggest that students drew on the latter as a basis for formulating their statement.

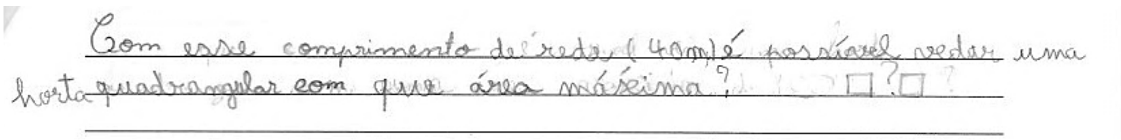
Concerning the first dimension (*Strategies*), the problems solved allowed for *multiple strategies*, as did 82% of the problems posed. In T1, students solved a problem involving the determination of a rectangle’s perimeter and area, which could be approached using different methods, such as trial and error or direct application of formulas. The same was true for the problems posed in T2.

However, unlike in T1, in T2 problems the possibility of presenting multiple solutions or employing creative approaches was absent. This suggests that students might still have been developing the ability to create problems that allowed multiple solutions. As such, when posing problems, many pairs replicated the solution almost entirely (Figure 6).

Table 5. Comparison between posed and solved problems

		T1 (PS)	T2 (PP)		T7 (PS)	T8 (PP)	
			n	%		n	%
Strategies	Multiple strategies	✓	9	82	✓	9	82
	Multiple solutions	✓	0	0	✓	2	18
Concepts	Creative solutions	✓	0	0	✓	2	18
	Other mathematical concepts	X	0	0	X	1	9
	Other geometry concepts	X	0	0	✓	6	55
Context	Real-world	✓	11	100	X	11	100
	Logical and plausible situation	✓	11	100	X	10	91
	Students' everyday experiences	X	0	0	X	10	91

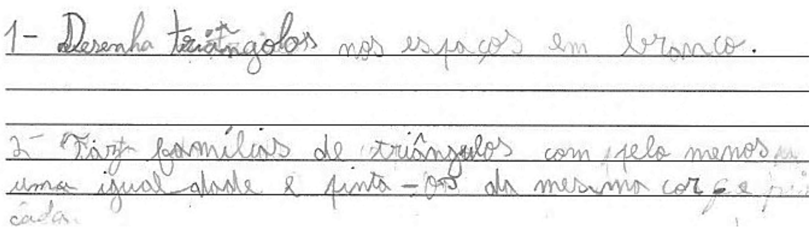
Figure 6. PP in T2, pair E


Com esse comprimento de rede (40m) é possível vedar uma horta quadrangular com que área máxima?

Translation: With that length of netting (40 m), is it possible to enclose a quadrangular net with how much maximum area?

In T7, students solved a problem about quadrilateral families, where different criteria, such as the positioning of sides or the angles' measures, enabled various classifications and, consequently, multiple solution strategies. This positively influenced T8, where the pairs incorporated the idea of multiple approaches to grouping triangles, such as pair H, who first asked to draw different triangles, allowing for a range of approaches, and then to create families of triangles using any criteria the solver chose, but sharing at least one common characteristic (Figure 7).

Figure 7. PP in T8, pair H


1- Desenha triângulos nos espaços em branco.

2- Faz famílias de triângulos com pelo menos uma igualdade e pinta-os da mesma cor

Translation: 1 – Draw triangles in the blank spaces.
2 – Create families of triangles with at least one equality and color them the same.

For the dimension *Concepts*, the absence of parameters related to the *mobilization of other concepts* in T1 and T2, which focus primarily on area and perimeter, suggests that in their first PP activity, pairs replicated characteristics present in the problem they had solved.

There is consistency in these aspects when comparing T7 and T8. Specifically on the application of *other geometric concepts*, T7, which involved classifying quadrilaterals, allowed students to apply a variety of geometric concepts. This approach continued into T8, with 54% creating statements that required recognizing various geometric properties in triangles. Some students who used measurement to solve the problem in T7 also incorporated measurements in the problem posed. For instance, pair K used the shapes' perimeter as a basis for both tasks (Figures 8 and 9), demonstrating a clear transfer of ideas from one task to the next.

Figure 8. Resolution of T7, pair K

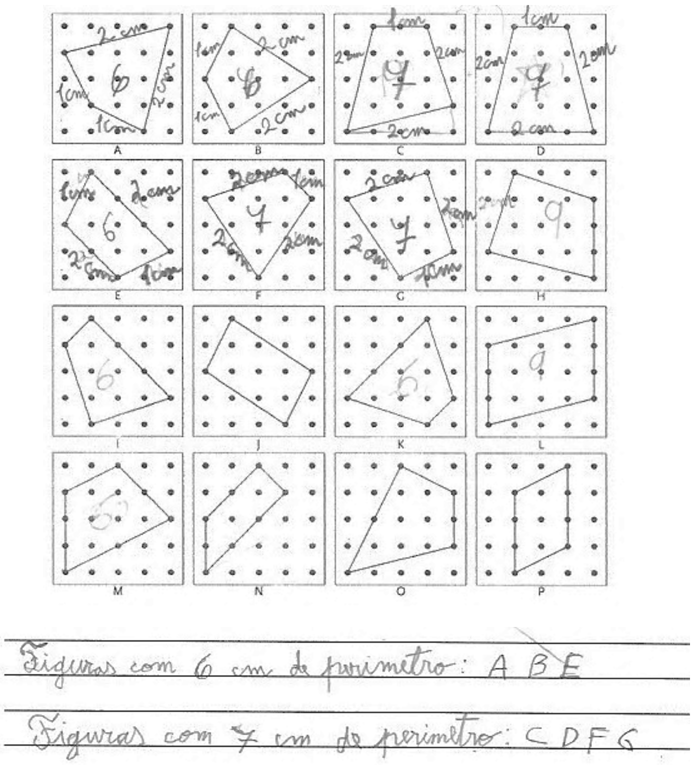
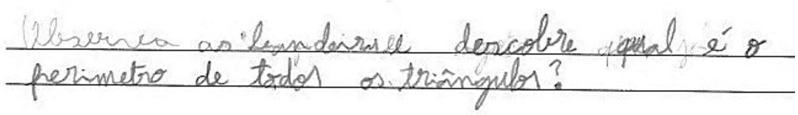


Figure 9. PP in T8, pair K



Translation: Observe the flags and discover the perimeter of all the triangles.

Concerning the third dimension (*Context*), the characteristics of the problem statements remained consistent from T1 to T2, with both considered *real-world scenarios* and *representative of a logical situation* (100% in both parameters for T2). Using real-world problems allowed students to contextualize mathematical problems in practical situations, which may have facilitated their understanding and resolution of tasks. This practical approach was maintained in PP as students formulated questions about constructing a vegetable garden with a specific fence length. The problem in T1 involved fencing a rectangular park, and in T2, the problems posed by students followed a similar pattern, showing a continued use of real-world contexts: "Rui wants to build a vegetable garden, but he only has a fence with a perimeter of 40 meters" (pair I). Similarly, we observed the same characteristics in T1 and T2 concerning *students' daily lives*. The initial problem involves designing a park that we did not consider part of students' day-to-day experiences, and students used the same context for the problems they posed.

On the contrary, in T7 and T8 there was a visible change in the problem statements contexts. T7 showed an absence of all three parameters, while T8 exhibited occurrences of more than 90% for all parameters. This difference was due to T7 not referring to a real-world situation, whereas T8 was explicitly framed around a concrete scenario. This contrast was expected and stemmed from the problem statements' characteristics.

5. Discussion and conclusion

The results of this study, conducted in the context of a teaching experiment focused on the interplay between PS and PP, aim to shed light on the evolution of characteristics of the problems posed by students and the relationship between the problems they posed and those they solved. In the following, we discuss these results, examine the study's limitations, and provide insights into future research.

5.1. Characteristics of the problems posed

The geometric nature of the problems posed and solved in this study likely influenced the results, as compared to more abstract mathematical areas; the spatial and visual characteristics of geometry may help students engage with concepts more concretely, promoting connections between solved and posed problems. This aligns with previous findings in that task format and content domain, such as geometry, play a significant role in shaping students' PP and PS abilities (Cai et al., 2013; Radmehr & Vos, 2020). The improvement in mathematical language, data richness, and solvability could therefore be partly attributed to the accessible and structured nature of geometry tasks from the teaching experiment that encouraged exploration and application of diverse strategies.

Throughout the teaching experiment, the quality of PP improved significantly across all evaluation criteria (Table 2), with over half of the pairs achieving the highest scores by the end. Significant progress occurred in *aligning problems with task prompt* and using *mathematical language*. For alignment, the total score increased from 19 points to 26. This reflects improved linguistic coherence and mathematical precision, as problems posed in T8 were clearer and better connected to task requirements than those in T2, which often exhibited ambiguities.

In *mathematical language*, scores shifted from predominantly low in T2 to higher in T8, with the total score rising six points. The improved use of precise and complete mathematical terminology is in line with Özgen et al.'s (2017) findings, where the exposure to structured problem-solving gradually enhanced the ability to employ precise language. The *quantity and quality of data* in problems also improved. While early tasks often lacked sufficient or relevant data, later ones demonstrated better-structured and logically complete information, enabling the creation of more complex problems. *Originality* saw progress but remained challenging. Many students continued to replicate previously solved problems or examples, showing reliance on familiar models rather than independently generating novel ideas. This result lines up with Sadak et al. (2022), who emphasized that structured environments might restrict novel ideas' exploration. This suggests that structured prompts may still limit creativity, even if they help make tasks easier in initial phases.

Solvability showed an improvement, with 91% of the problems in T8 deemed solvable, compared to 64% in T2. Initially, some problems were unclear or unsolvable due to lacking necessary data. In contrast, almost all the statements produced for T8 included sufficient and appropriate information, reflecting an even better understanding of conditions for solvable problems.

These results seem particularly relevant because, unlike other PP teaching approaches (e.g., Papadopoulos & Patsiala, 2023), this study encouraged diversity by having students create problems from both given situations and their own experiences with specific math topics. This improvement suggests that with practice, students become more proficient at creating high-quality problems. However, pedagogical guidance remains essential to encourage greater independence and creativity in problem formulation.

It is important to consider that, although the results indicate significant progress in the originality and quality of the problems posed in T8 compared to T2, the openness of T8 may have played a key role in this progress, as it allows greater creative freedom. Thus, this highlights the need to carefully consider the impact of task format on the development of PP competencies (Cai et al., 2013).

5.2. Relationship between problems posed by students and those solved

The comparison between solved and posed problems shows that students often drew on their experience with previous tasks to create new ones, evident in the similarities of task characteristics. Key influences included *multiple solution strategies*, *real-world contexts*, and *logical and credible situations*. Particularly in T2 and T8, most pairs created problems allowing for multiple solutions, mirroring those in T1 and T7.

Results suggest that features emphasized at one stage of intervention often persisted in later tasks, even without explicit reinforcement. For example, all pairs in T2 created *real-world, logical, and credible problems*, like the one in T1. Most pairs retained these features in T8, despite their absence in T7, which corroborates the idea that real-world contexts are easier for younger students to integrate (Palmér & Bommel, 2020).

Another finding is the shift in concept application across tasks. From T1 to T2, students often replicated key ideas like perimeter and area, leading to straightforward, less original problems. In contrast, they used a broader range of geometric concepts, such as angles, sides, and measurement relationships, in T7, which contributed to broader problem diversity in T8. They incorporated different types of triangles and challenges requiring more complex reasoning, confirming that diverse and progressive tasks can broaden students' mathematical repertoire. This aligns with Silver and Cai (1996), who argued that students' PP can reflect rich mathematical connections when exposed to diverse tasks.

Initially, students often replicated elements from previous tasks, limiting originality. Over time, through repeated PS and PP, they gained autonomy, creating more original problems, distinct from those in class or textbooks. In contrast with other studies which used explicit replication instructions (Pálmer & Bommel, 2020), our study's design paired PS and PP tasks that only matched in mathematical topics, challenge level, and solution processes. This approach promoted a more natural exploration of the relationship between these activities, fostering an autonomous transition between them.

The students' progress underscores the importance of consistent and extended practice in developing PP skills. These results align with Sadak et al. (2022), emphasizing the value of integrating creative problem-posing and solving tasks in mathematics education to enhance *creativity dimensions* such as fluency, flexibility, and originality.

These findings illustrate how PS tasks directly influenced subsequent PP and how the richness and complexity of initial tasks impacted the quality and originality of later ones.

5.3. Limitations and suggestions for future studies

Despite yielding relevant results, this study has limitations. Students did not solve their own problems, limiting insights into how they manage self-created challenges. While this avoided discouraging complex formulations, future research could examine how solving their own problems impacts strategy and reasoning. The short intervention contrasts with longer ones (e.g., Papadopoulos & Patsiala, 2023), which may better capture students' progress with complex problems, varied strategies, and broader content. A further limitation was the differing openness of tasks T2 and T8. T2 was more closed, requiring students to formulate a problem based on a specific solution, while T8 allowed more creative freedom, possibly influencing originality and strategy variety. Thus, improvements from T2 to T8 may partly reflect task design. Real-world prompts also shaped problem contexts. Future research should ensure more consistent task design or investigate how openness impacts PP learning.

As students initially struggled to pose problems allowing *multiple strategies*, future studies could explore how to support this skill. Although PP is critical in understanding mathematical concepts, it does not inherently foster creativity, hence the need to value both creative PP and PS (Sadak et al., 2022). Less prescriptive tasks can stimulate creativity and diversity, while structured environments with guided practice (Hiebert et al., 1996) are vital for building deep math skills. Future research could explore balancing pedagogical guidance with students' autonomy to foster both originality and thorough mathematical understanding in PP.

5.4. Final remarks

This study, based on a teaching experiment alternating PP and PS tasks aligned with the primary mathematics curriculum, demonstrated that even a short intervention can significantly improve the ability to structure solvable mathematical problems. These problems showed proper sentence construction, orthographic accuracy, and alignment with learning objectives.

Findings showed that prior experiences with PS tasks directly influenced subsequent students' PP. They often reused features from solved problems, and some characteristics persisted even without explicit emphasis. This suggests they can retain and apply key aspects to enhance their PP skills, though reliance on familiar structures sometimes limited originality.

The study shows that solved problems shape new ones by internalizing patterns (Brown & Walter, 2005; Hiebert et al., 1996), helping students apply concepts across tasks. Still, fostering diversity in problem statements is challenging. Thus, the study underscores the benefits of diverse PP experiences in enhancing the capacity to create meaningful, contextually appropriate mathematical problems, highlighting the value of integrating such tasks into mathematics teaching.

Acknowledgment

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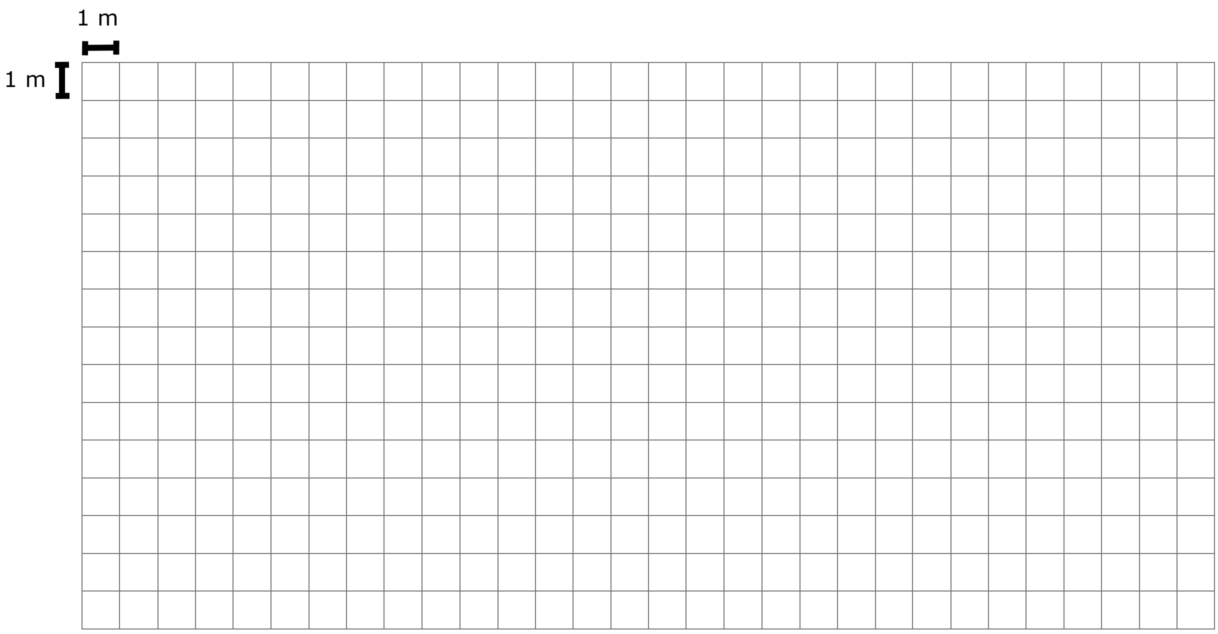
APPENDICES

Appendix I – T1

Teresa wants to build a rectangular park for her dog Cereja and intends to use a 36-meter-long fence to enclose it.

1. If she uses all the fence, what different shapes could the park be?
2. Of those, which one will have the largest area? And the smallest?
3. Of all of them, which one will be the best for Cereja to run in?

Draw your designs on the grid paper and explain your thinking.



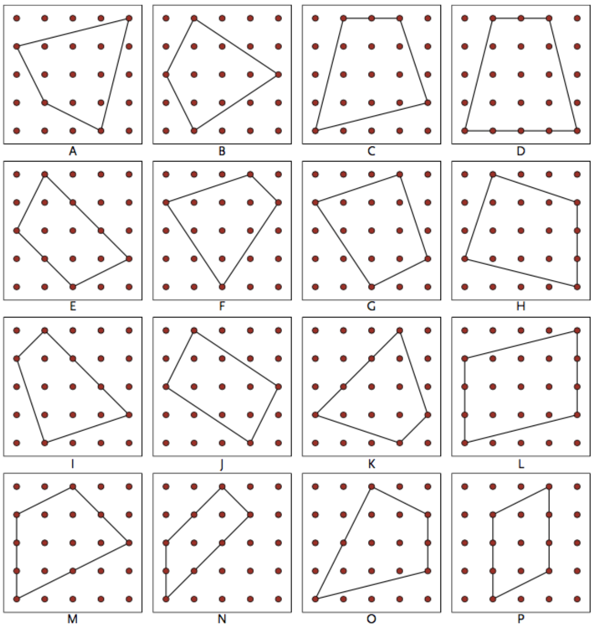
Appendix II – T2

Pose a mathematical problem with the following solution:

With this length of fence, it is possible to enclose a square vegetable garden with a maximum area of 100 square meters.

Appendix III

Observe the quadrilaterals.



Sort the quadrilaterals into families, according to their characteristics.

Explain your reasoning

Appendix IV – T8

Create a word problem based on the dot grids provided and the following scenario:

Students in a class want to make several triangular flags with different shapes.

∞

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Conectando resolución y formulación de problemas: Estudiantes de tercer grado aprendiendo a plantear problemas en Geometría y Medición

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Este estudio explora la relación entre la resolución de problemas (RP) y la formulación de problemas (FP) en una clase de tercer grado, centrándose en el dominio de Geometría y Medición. A lo largo de ocho lecciones diseñadas como parte de un experimento de enseñanza, alternamos tareas de RP y FP alineadas con el currículo de Matemática de Portugal, con el propósito de observar cómo los alumnos de 8 años usan su experiencia en RP para generar nuevos problemas.

Para evaluar la calidad de los problemas formulados, adaptamos un instrumento donde fueron considerados siete criterios: uso del lenguaje matemático, gramática y expresión, adecuación a los objetivos de aprendizaje, cantidad y calidad de datos, originalidad, alineación con las indicaciones de la tarea, y solvencia. Los resultados muestran que, al principio, los estudiantes formulaban problemas inspirándose en los previamente trabajados en clase, que funcionaban como modelos; sin embargo, a lo largo de las lecciones, comenzaron a crear problemas más originales, lo que evidencia un desarrollo en su capacidad de FP. Las tareas de FP no solo les ayudaron a mejorar en la formulación de preguntas matemáticas, sino también a usar un lenguaje matemático más preciso, mejorar la gramática y la expresión, y crear problemas que eran matemáticamente sólidos y resolubles. En particular, se observó que los estudiantes mejoraron significativamente en el uso del lenguaje matemático y en la calidad de los datos presentados en los problemas, lo que refleja un progreso en sus competencias matemáticas generales.

Además, se analizó cómo las tareas de RP, en las cuales resolvían problemas matemáticos, influían en la formulación de problemas durante las tareas de FP. La interacción entre estas dos actividades resultó ser crucial para el desarrollo de habilidades de pensamiento crítico y creativo. A través de esta experiencia, se destacó la importancia de un diseño de tareas que fomente la reflexión sobre las soluciones encontradas, para que los estudiantes puedan generar problemas nuevos basados en sus propias soluciones, contribuyendo así a su autonomía en el aprendizaje matemático.

Finalmente, el estudio demuestra que, aunque inicialmente los estudiantes replicaban estructuras de problemas resueltos, gradualmente fueron capaces de crear problemas más complejos y personalizados. Este desarrollo sugiere que la integración de tareas de FP como un objetivo de aprendizaje puede contribuir positivamente al desarrollo de la creatividad matemática de los estudiantes y su capacidad para abordar y formular problemas en el ámbito de la educación primaria.

En suma, el estudio muestra que la articulación entre la resolución y la formulación de problemas, apoyada en un diseño cuidadoso de tareas, constituye una vía eficaz para integrar la FP como objetivo de aprendizaje en la enseñanza de las matemáticas.