

## **Competencies in understanding statistical information in primary and secondary school levels: An inter-cultural empirical study with German and Colombian students**

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**Competencias en la comprensión de la información estadística en educación primaria y secundaria: Un estudio empírico inter-cultural con estudiantes alemanes y colombianos.**

### **Resumen**

*Este artículo está dedicado a investigar la validez de un modelo de competencia jerárquico a través de un análisis de los niveles correspondientes en primaria y secundaria en Alemania y en Colombia. Los resultados confirman el modelo de competencia jerárquico ya establecido en trabajos anteriores. El estudio que aquí presentamos tiene dos componentes novedosas: analiza la validez del modelo en primaria y es intercultural. Los resultados obtenidos por los estudiantes sin embargo sugieren perfiles específicos de subgrupos especiales de los alumnos analizados. El estudio intercultural es meramente exploratorio pues las muestras incluidas en Colombia y Alemania no eran representativas. Los resultados apuntan a nuevas preguntas y motivan otras investigaciones interculturales en un futuro.*

**Palabras clave:** Modelo de competencia jerárquico; niveles de competencia; escuela primaria; escuela secundaria; estudio intercultural.

**Competencias na comprensao da informacao estadística na educaçao primária e secundária: Um estudo empírico inter-cultural con estudantes alemanes y colombianos.**

### **Resumo**

*Este artigo se dedica a investigar a validez dum modelo de competencia hierárquico através duma análise dos níveis correspondents em primaria e secundaria em Alemanha e em Colombia. Os resultados confirmam o modelo de competencia hierárquico ja estabelecido em trabalhos anteriores. O estudo que aqui presentamos tem duas componentes novas: analisa o modelo em primaria e é intercultural. O estudo intercultural é meramente exploratorio pois as amostras consideradas em Alemanha e em Colombia nao eram representativas. Os resultados trazem novas perguntas e motivao outras investigacoes para um future.*

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**Palabras clave:** Modelo de competencia hierarquico; niveles de competencia; escuela primaria; escuela secundaria; estudio intercultural.

**Competencies in understanding statistical information in primary and secondary school levels: An inter-cultural empirical study with German and Colombian students**

**Abstract**

*This paper is devoted to examining the validity of a hierarchical competency model of statistical literacy by assessing the corresponding competency levels of primary and secondary students from Colombia and Germany. The results replicate prior findings and confirm the hierarchical competency model. The study has two novel components with respect to previous work on competency levels in statistical literacy: it examines elementary school performance and it includes an intercultural comparison. The competency scores reached by the students, however, suggest specific profiles of sub-groups of learners included in the study. The intercultural study is clearly exploratory since the samples both in Colombia and in Germany were not representative. The results point to interesting new questions and motivate further intercultural research.*

**Key words:** Hierarchical competency model; levels of competency; elementary school; secondary school; intercultural study.

**Compétences dans la compréhension de l'information statistique dans l'éducation à l'école primaire et secondaire**

**Résumé**

*Cette contribution est dédiée à étudier la validité d'un modèle de compétence hiérarchique à travers d'une analyse des niveaux correspondant aux étudiants de l'école primaire et secondaire en Colombie et en Allemagne. Les résultats confirment le modèle de compétence hiérarchique déjà établi dans des travaux antérieurs. L'étude a deux nouvelles composantes par rapport aux travaux antérieurs: elle examine la performance à l'école primaire et elle inclut une comparaison interculturelle. Les résultats en compétence atteints par les étudiants suggèrent des profils spécifiques dans des sous-groupes particuliers. L'étude interculturelle est exploratoire, car les échantillons en Colombie et en Allemagne n'étaient pas représentatifs. Les résultats dénotent des nouvelles questions intéressantes et motivent des futures recherches interculturelles.*

**Paroles clés:** Modèle de compétence hiérarchique; niveaux de compétence; école primaire; école secondaire; étude interculturelle.

## **1. Introduction**

Competencies in understanding statistical information are necessary for social participation of citizens and hence should be fostered in the mathematics classroom. Assessing such competencies of learners can support the design of learning opportunities which fit to the learning needs of students. As in prior research, a model of the competency “using models and representations in statistical contexts” has been developed and tested empirically in the context of secondary education. There has to date been no specific cross-cultural evidence neither about the hierarchical structure of the model nor about possible intercultural differences in competencies. Moreover, competencies of primary students have to date not been assessed on the base of this competency model. Empirical evidence about the level of competencies students can reach in different educational contexts is hence at the centre of interest, even beyond the evaluation of e.g. different curricular conceptions and characteristics of the mathematics classroom.

Consequently, as a first step, this study concentrates on assessing primary and secondary students from Colombia and Germany. The results replicate prior findings and support the hypothesis of the hierarchical structure of the competency model. The competency scores reached by the students however suggest specific profiles of the sub-groups of learners included in the study. Hence, this research broadens the evidence base related to competencies of students, including students at the primary level, and furthermore invites follow-up research about instructional factors that impact the development of statistical literacy.

In the following second section, we give an overview of the theoretical background of the study and in particular of a competency model, which was implemented in a corresponding test instrument. As a base for the cross-cultural comparisons, the third section presents context data about schooling and cultural background of the groups of students who participated in this study. Against this background, section 4 leads to the research questions. After describing the design and the sample in section 5, we report about results in section 6, which are discussed in the concluding seventh section.

## **2. Theoretical background: Statistical literacy**

Research on statistical literacy has received growing interest over the past 30 years, as competencies in this area have become considered crucial to responsible citizenship in modern information societies: Newspapers, magazines or online resources as well as medical brochures, insurance booklets and political advertisements confront readers with statistical information and diagrams. With the help of modern technologies, complex data is extracted from real world contexts and presented to the public, often without much simplification and details about the process of data gathering or analysis. Decisions on the social and political levels are regularly informed by such representations of data. Taking part in the democratic decision process as a responsible citizen thus requires abilities of interpreting and of critically evaluating such statistical information (Wallman, 1993; Watson, 1997; Watson & Callingham, 2003; Wild & Pfannkuch, 1999).

When critically evaluating data given in diagrams, tables, or other statistical representations, mathematics-related modelling processes are often necessary. We would like to use the example given in Figure 1 for highlighting this point. The example most likely leads to a multi-step evaluation of the diagram that can be described by a modelling cycle (e.g. Blomhøj & Jensen, 2003; Blum & Leiß, 2007). By making sense of this diagram, mathematics comes into play. In Figure 1, the additional task on the right aims at emphasising such activities of mathematical modelling. As a first mathematical model, readers might try to interpret the length of the vertical lines as the representation of the given values. This model yields results which can be validated with the data given in the diagram.

After discarding this model, given the evidence, comparisons between the values (ranging from simple comparisons of size to a proportional reasoning model, e.g. “12.6 millions is about the double of 6.6 millions”) may lead to the insight that the horizontal distances better reflect the given values. The model of the inverted number line will then enable the readers to localize the missing origin zero on the right side of the diagram. Again, having constructed this mathematical model, a problem solver might then check how it fits to the context, i.e. to the given data or to additional information

about the situation context the data stems from. As this example may hence illustrate, modelling can play a crucial role for competencies in understanding representations of statistical information (cf. Engel & Kuntze, 2011; Kuntze, 2010).



Find out about how the data is represented in this diagram. Please describe how the diagram “works”. Is the data represented correctly and/or to what extent is the representation misleading for readers? Please suggest how the diagram can be improved (if appropriate).

Figure 1. Problem about a representation showing the numbers of members of sects (Diagram taken from the magazine “Soll und Haben”; SZ-Magazin 51/2006, p. 22ff, text translated by the authors).

Beyond representations in diagrams, a common aspect of data-related modelling has to do with the overarching idea of reduction of information (Kröpfl, Peschek & Schneider, 2000; Wild & Pfannkuch, 1999). Data extracted from real-world contexts might be difficult to oversee – the bare quantity of data often hinders us to identify tendencies, trends, differences or interdependencies contained in the data. For gaining an overview, statistical models and algorithms can be used, e.g. a mean or the mean deviation from the mean can be calculated for given lengths of bananas or the incomes of citizens. However, calculating such values reduces the initial information – this loss of information has to be taken into account when interpreting the derived values.

Dealing with statistical data often means navigating between the extremes of “a lot of information without overview” and “a good overview without a large part of the information”, in a spectrum like shown in Figure 2. For example, if only a mean value is given, it may be difficult or impossible to make a statement about the distribution of the data, about the interrelatedness of variables or about the frequency of particular values. Hence, learners should be aware of this phenomenon, we will refer to it in the following as the idea of reduction.



Figure 2. Spectrum linked to the idea of “manipulating data by reduction of information (cf. Kröpfl, Peschek & Schneider, 2000)

A further overarching idea plays a fundamental role for abilities of evaluating data and data-related modeling: dealing with statistical variation (Watson & Callingham, 2003; Watson, Kelly, Callingham, & Shaughnessy, 2003) is often a key factor when having to judge on e.g. differences and tendencies contained in data. For example, does a value indicate a meaningful change in a series of measurements or should it only be considered as a non-significant statistical deviation (cf. Engel & Kuntze, 2011)? Can a value be looked at as fixed or determined or do we have to assume that another measure would probably give a slightly different value?

For describing and dealing with statistical variation, we can use mathematical models such as for example confidence intervals, standard deviations, probabilistic models, etc. Conversely, modelling activities in statistical contexts can very frequently be seen as connected with the overarching idea of dealing with variation.

The above ideas were integrated in a competency model by Kuntze, Lindmeier and Reiss (2008), that reflects the competency of using models and representations in statistical contexts (Figure 3). The model is based on the assertion that data-related reading -as an enlarged metaphor compared to Curcio’s (1987) approach of “reading in data” - combines requirements of manipulating data through reduction with requirements of dealing with statistical variation: Data-related reading is based on representations of data and it is often only possible with the use of statistical models which facilitate manipulating the data by reducing and/or dealing with variation contained in the data.

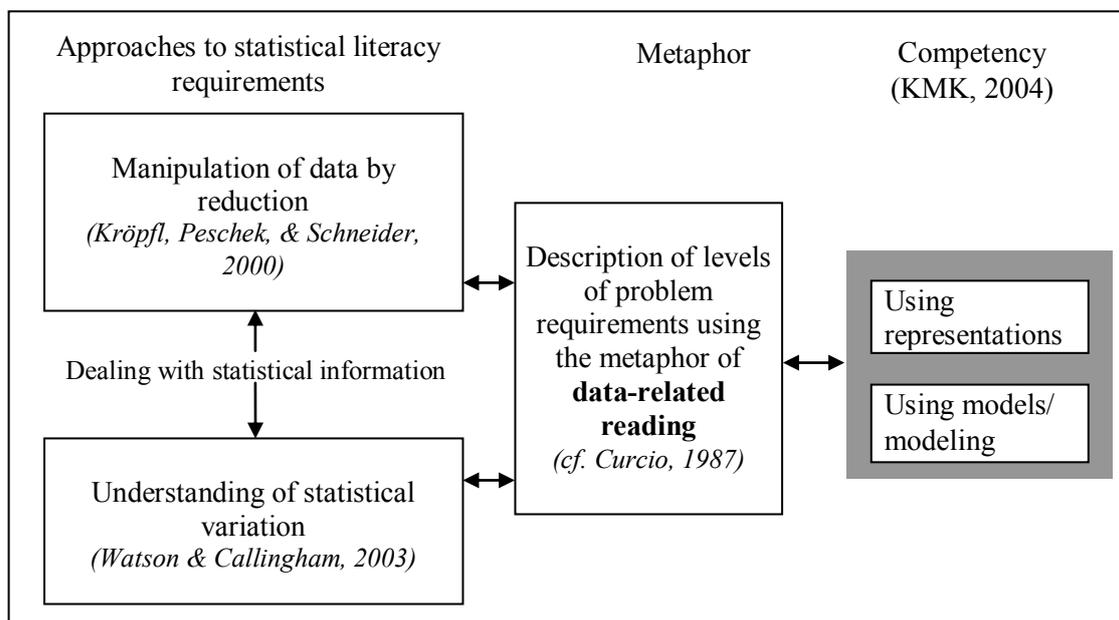


Figure 3. Aspects of requirements in the domain of statistical literacy linked to the competency “using models and representations in statistical contexts” (Kuntze, Lindmeier, & Reiss, 2008, p. 2)

Moreover, as stated above, data-related reading does not only requires the use of representations or changing representations, but also of modelling activities. As a result of these theoretical requirements (more detailed presentation of the theoretical background of the model is given in Kuntze, Lindmeier & Reiss, 2008; Kuntze, in press), this competency then reflects both the aspects of modelling and using representations, as understood in the German standards (KMK, 2004), for the specific content area of problems in statistical contexts.

Based on theoretical considerations in which the ideas of dealing with variation and reduction play a role, the model by Kuntze, Lindmeier and Reiss (2008) affords describing the competency of students in a hierarchical way. Table 1 gives a description of five levels of competency, in the enlarged version of Kuntze, Engel, Martignon and Gundlach (2010) and Kuntze (in press). In several empirical studies based on test instruments designed according to this model, the competency model has shown to be hierarchical and to conform to a one-dimensional Rasch model (Kuntze, Lindmeier and Reiss, 2008; Kuntze, Engel, Martignon and Gundlach, 2010).

Table 1. Competency model for using models and representations in statistical contexts (Kuntze, in press; Kuntze, Engel, Martignon and Gundlach, 2010); in bold: aspects which difference each level in comparison with the lower levels.

Level I	<b>One-step use</b> of a representation or work within a given model (e.g. reading a given value from a diagram, completing a given diagram for a given table)
Level II	<b>Two- or multi-step use</b> of representations or changing between two given models (e.g. comparing data including a transformation step or a mathematical concept)
Level III	Multi-step use of representations including the <b>use of a non-given model</b> (e.g. own modeling activities supporting a cumulative interpretation of data given in diagrams)
Level IV	Multi-step use of representations and/or use of a non-given model requiring an adequate <b>dealing with statistical variation</b> (e.g. own modeling activities based on diagrams that require dealing with statistical variation)
Level V	Multi-step use of representations and use of a non-given model that requires dealing adequately with statistical variation and <b>modeling/constructing mentally a non-given set of data</b> (e.g. modeling possible underlying data when judging on given statements)

A newspaper's headline:  
**"About 80.000 households use heating oil, about 40.000 use petroleum gas for heating."**

The headline is accompanied by the following diagram:



Heating oil Petroleum gas

Why does this diagram represent the situation in a wrong way?

Figure 4. Sample problem for level of competency III (cf. Kuntze, Lindmeier & Reiss, 2008)

Figures 4, 5, and 6 display three sample problems from the test instrument, representing three different levels of competency. The item in Figure 4 (cf. Kuntze, Lindmeier & Reiss, 2008) corresponds to level III competency: it requires the use of a non-given model, as the data presented in the diagram have to be interpreted against the background of an area model shown and its measure. Using such a model affords a correct interpretation of the relative size of the symbols displayed in the diagram against the background of the given numbers. The model of measuring the area covered by the symbols is not given to the students.

In addition to using a non-given model, the problem in Figure 5 requires dealing with statistical variation. For understanding the data in the diagram and answering the question, the students need to make sense of non-determinist deviations. The doctor mentioned in the task might argue in terms of mean or extreme values. However,

dealing with the variation contained in the data by an appropriate model plays a crucial role for the arguments that the learners have to find.

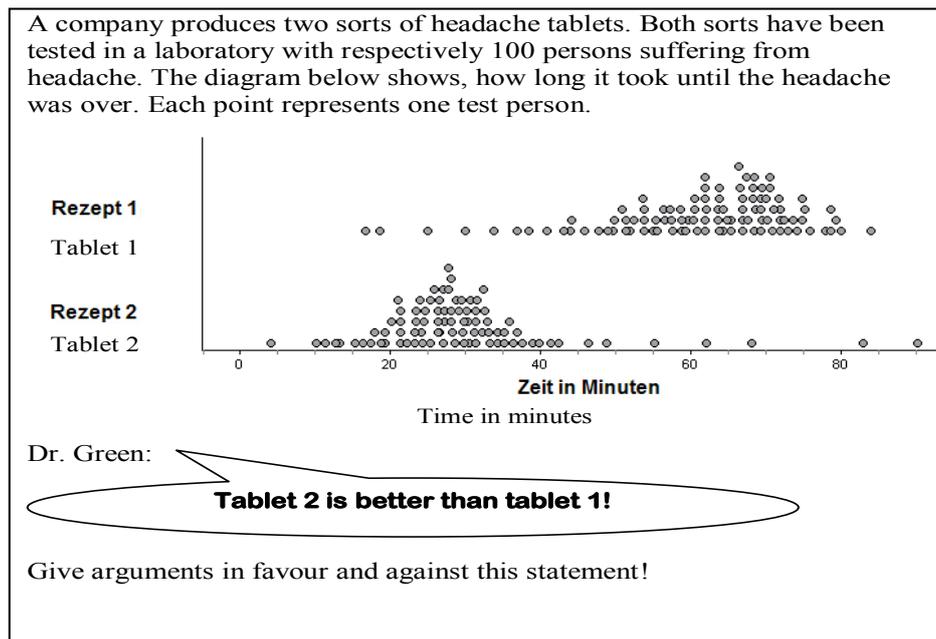


Figure 5. Sample problem for level of competency IV (cf. Kuntze, Lindmeier & Reiss, 2008)

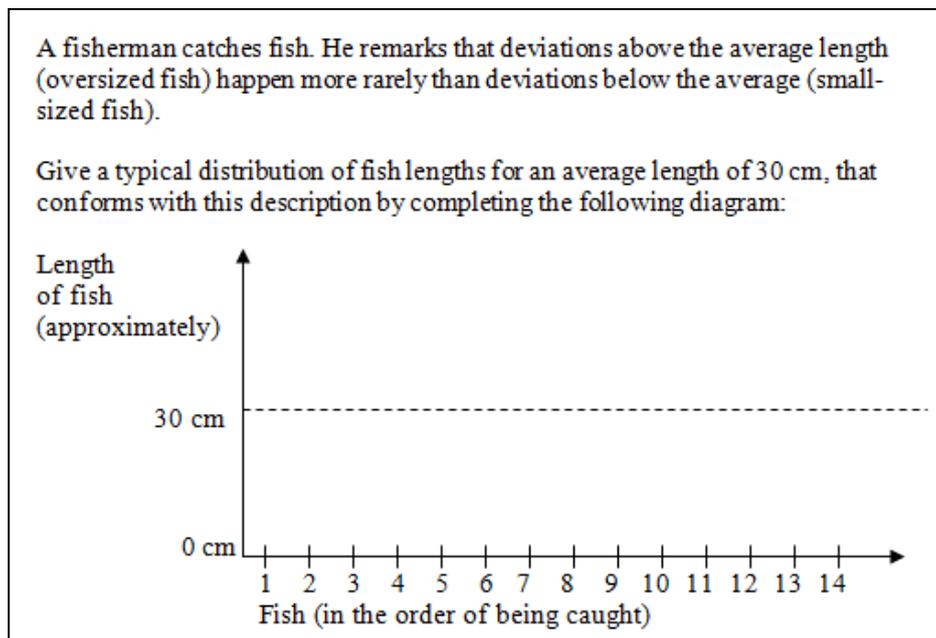


Figure 6. Sample problem for level of competency V.

Situated on the highest level of competency V, the problem in Figure 6 requires the modelling and mental construction of a non-given data set according to the given properties. This presupposes also abilities of dealing with statistical variation as the distribution will typically not be ordered e.g. according to the length of the fish. Again, the ability of using mathematical models like the average of given values is a necessary precondition for solving the problem.

In line with the theoretical expectations, prior research with secondary and university students has shown that the empirical item difficulty of the competency levels increased with the levels of competency (Fröhlich, Kuntze & Lindmeier, 2007; Kuntze, Engel, Martignon & Gundlach, 2010; Kuntze, Lindmeier & Reiss, 2008).

### **3. Statistical literacy, schooling and culture**

As democratic values apply on a cross-national level, developing statistical literacy is important for all countries. Democracy and global problems require responsible “world citizens” who can participate in social discourse in the sense that they have the necessary competencies not to depend from the interpretations and judgments by third parties when having to make sense of statistical information. Knowledge and skills to reason adequately with data are an important prerequisite for the functioning of democracy in our mass societies. Active democrats and concerned citizens need skills in reading statistics and charts as well as in interpreting and critically evaluating data to understand the pressing social and political issues of our time and to be actively involved in policy-making and decision-making processes or to monitor progress of human development (Engel, 2013).

Unfortunately and mainly for reasons of curricular traditions, most countries had for many years been reluctant to allot a significant portion of school hours to data analysis and statistics. During the past decades this has been changing (Batanero, Godino, Valecillos, Green, & Holmes, 1995), and many national standards today see building up competencies in data analysis and statistical reasoning as mandatory. One important reason for this change, in Germany, for instance, has been the achievements of school students in international student assessments such as PISA (OECD, 2003) on tasks belonging to the area of “uncertainty”.

In some countries, like Germany, the specific lack of competency of students in solving such tasks was a drawback to their overall performance (e.g. Deutsches PISA-Konsortium, 2004). The performance of the students in Germany in the PISA tests of the last 10 years has been average or slightly above the average. As a reaction to these results, reforms have been carried out under the influence of the standards by the National Council of Teachers of Mathematics (NCTM, 2000), which have been adapted to the German educational context (KMK, 2004). As a result of these reforms, school curricula in all States of Germany include statistical contents over the several phases of schooling, beginning at the primary level and emphasising competencies in statistical reasoning throughout secondary school of all school types. However, teachers are often not optimally prepared to teach these contents (cf. e.g. Batanero et al., 2011, Kuntze & Kurz-Milcke, 2010), as these contents are still relatively new in the curriculum, and there is the danger that tasks in the textbooks are interpreted in a purely algorithmic way, without putting an emphasis on the ideas behind statistical literacy such as those we have introduced in the previous section.

The Latin American countries are usually at the extreme end of the PISA ranking, as the PISA reports in 2000, 2003, 2006 show. Some of these countries lacked for many years the implementation of general curricula released by a central organization. Until recently, Colombia, for instance, did not have national programs for any of the main subjects. In the past years significant efforts have been made for improvements on the enrolment rates of the students, but the results are not very satisfactory, when looking at the quality and equity of education (Barrera-Osorio, Maldonado, Rodríguez,

2012). Whereas in Germany the public school system is predominant and educates more than 90% of children in the Federal Republic, the school system in Colombia is divided into two large subclasses, public and private schools. About 16% of students attend private schools and the rest attend public schools<sup>1</sup>. Between these subclasses of the school system, many context factors differ.

Private schools in Colombia are considered to be the best schools, and among the private schools those with bi-national bilingual programs are socially regarded as among the best. This is probably due to the fact that such schools often follow programs from European or North-American countries. Moreover, social selection effects are very likely to have an impact on the level of the students' recruitment for private schools. Hence, such private schools offer the opportunity of assessing competencies of students in a school environment that is marked by the goal of keeping up with best-practice standards in Europe, or the U.S. At the same time, the students in most cases come from a different cultural background than e.g. students in Germany.

In contrast, for the German sample, we chose at a more average level: students at public primary schools in Germany have not undergone any selection process. At the secondary level in south-western Germany, there are five types of secondary schools: Academic-track schools, technical-track schools, two types of schools for lower achieving students and schools for students with special needs. Technical-track schools (the so-called "Realschule") have a broader range of emphasis on pupils of a medium achievement level and students end their studies with a final examination ("Mittlere Reife") after grade 10. Students from technical-track secondary schools thus may represent an average level of achievement compared to all German secondary students.

Practitioners report that at German technical-track schools, contents related to statistics still do not have a high priority in the classroom and are sometimes somewhat "neglected" by teachers. Consequently, in different school cultures, as defined by their setting in countries, school types, socio-economic backgrounds, curricular circumstances and other context aspects we expect that competencies in the area of statistical literacy could be very different. Moreover, little is known whether the levels of competency as introduced above are really hierarchical not only according to theoretical considerations but also empirically for samples from different cultural settings. This study hence aims to provide first cross-cultural empirical evidence.

### **Statistical literacy in primary school level**

Moreover, the competency model has to date not been extended to primary school students. Again, and to a different degree, exploring whether the competency model can be considered as valid for primary school students of grade 4 is an interesting question when it comes to evaluating the domain of applicability of the competency model. In general, we admit that especially for learners on the primary level, adaptations may have to be made, as far as the test design is concerned.

In prior work (Fröhlich, Kuntze & Lindmeier, 2007; Lindmeier, Kuntze & Reiss, 2007; see also Kuntze, Lindmeier & Reiss, 2008) a test version comprising of problems at the competency levels I-III has been administered to academic-track

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<sup>1</sup> These data stem from a site of the Colombian Ministry of Education (<http://menweb.mineducacion.gov.co/seguimiento/estadisticas/>)

students in the 5th grade. The results suggest that 4th-graders at primary schools could also perform successfully on the test items. However, as there is a selection according to achievement after the 4th grade in Germany, little is known about the competency of students at the primary level.

#### **4. Research questions**

As emphasized above, cross-cultural studies about competencies in the area of statistical literacy respond to a research need: There is an obvious need of assessment instruments that can be used in comparative studies across cultures. Consequently empirical research related to the competency “using models and representations in statistical contexts” and its hierarchical structure as verified by a corresponding test instrument with German students should be extended to investigations into other school cultures.

In line with the research needs described above, we chose to test students at two age levels:

- At the primary level, comparing fourth grade students in Germany and Colombia;
- At the secondary level, comparing ninth grade German technical track students with a corresponding sample (as far as mathematical curricula are concerned) of secondary students in Colombia.

Responding to the research need described in the previous section, this study focuses on the following research questions, following an exploratory approach that takes into account the cross-cultural design:

- Do the data support the hypothesis that the competency model of “using models and representations in statistical contexts” is hierarchical also across cultural framing and variation in curricula?
- Is this hierarchy hypothesis empirically valid for students at the primary level?
- Which competencies in the area of using models and representations in statistical contexts do German and Colombian secondary students have?
- Which competencies in the area of using models and representations in statistical contexts do German and Colombian primary students have?

#### **5. Sample and methods**

For the purpose of this study, which consists in exploring the stability of the competency model with respect to culture and in gathering first evidence about the competencies of students in educational contexts different from Germany, the comparison with Colombian students at a private Italo-Colombian<sup>2</sup> school<sup>3</sup> has several advantages: Firstly, the Colombian and Italian characteristics of the school system represent a different cultural background, which is not as different from Germany as

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<sup>2</sup> It is worth mentioning that although the school follows a double program (Italian and Colombian), only around 10% of the students are of Italian origin.

<sup>3</sup> It is classified as “very superior” according to the rankings provided by the ICFES, the official government institution in charge of educational issues.

e.g. compared to East Asian countries. Secondly, even if social selection effects plays an important role, seizing the opportunity to compare with students from a private school in Colombia makes it possible to look rather at aspects related to mathematics instruction and its cultural framing, than only documenting deficits that may be due to a school system in non-optimal circumstances: For this purpose, it is useful to have a sample that affords expecting relatively high competencies. Thirdly, as primary and secondary schools at the Italian School in Colombia (Bogotá) are part of the same school, the two sub-samples (primary and secondary students in each country) correspond to a relatively similar cultural and institutional context, respectively. Of course, the Colombian school is not representative of the mean educational level of the country. Furthermore, the Italian school in Bogotá is a bilingual school and it combines both Colombian and Italian programs. The Italian programs curricula are defined as mandatory by the Italian Ministry of Education.

The test was administered both to German and Colombian students. In Germany, the sub-samples consisted of 385 fourth-graders (mean age 10.03; SD=0.61; 191 girls and 193 boys, 1 without data) and 549 ninth-graders (mean age 15.36; SD=0.71; 255 girls and 293 boys, 1 without data). The German students came from national schools in small-town and urban areas in Baden Württemberg, the state where the Ludwigsburg University of Education is located. The social environment of the schools may also be characterized as “middle class”.

In Colombia, 78 primary students (mean age 10.31; SD=0.52; 46 girls and 32 boys), and 78 secondary students (mean age 14.26; SD=0.59; 43 girls and 35 boys) were assessed. As described above, we assert strong social selection effects for the Colombian school so that the sample is not typical for Colombia as a country. We thus expect a relatively high level of competency of the students at the Italian school in Bogotá. However, the cultural environment and daily life conditions of the school children have to be considered as clearly different, given the setting in the two countries.

The choice of the age groups was done according to an analysis of curriculum and time of schooling – in particular the comparison of the curricula and the numbers of mathematics lessons per week in the lower secondary grades suggested to consider 8th-graders in Colombia as counterparts of the German ninth-graders. In fact, the mathematical topics covered by the 8th-graders in Colombia coincide with those covered by the 9th graders in Germany.

Based on prior research related to the test instrument by Kuntze, Engel, Martignon and Gundlach (2010) which is a modified and enlarged version of the test by Kuntze, Lindmeier and Reiss (2008), this study uses mainly quantitative methods.

For the two age groups, respectively, two versions of the test were used. The secondary students were administered a larger version of the test, comprising problems on all five levels of competency, whereas the primary students were asked to fill in a reduced version of the instrument, in which there were problems on the levels of competency I up to III only (cf. Table 1). This reduced version was based on experiences with grade-5 academic-track secondary students (Fröhlich et al., 2007) – the evidence of this prior research suggested that no ceiling effect would be observable for a test version focusing on the levels of competency I up to III.

As a small number of the tasks were considered to be not transferable one-to one in a translated version with respect to culture and the situation contexts mentioned in

these problems, the Colombian version of the test did not contain all problems of the German instrument. The comparisons presented below thus focus only on the tasks used for both German and Colombian students<sup>4</sup>.

In each country, the students were given a test during their school time to be filled in individually. They had about one hour for completing it. According to the experience in prior studies, this means that the time was largely enough for completing all of the test. The assessment was confidential and the names of the students were not recorded.

## 6. Results

We start by presenting sample answers of the students related to the problems presented in Figures 4, 5 and 6 above.

Figure 7 shows five answers by primary students for the problem in Figure 4, in which the students were asked why the given diagram represents the situation in a wrong way. Even though some answers show difficulties in linguistic expression, they were coded as correct if the evidence suggested that a correct mathematical model had been used to provide a reason as required by the question. For example, the first three answers were coded as correct. All of them make implicit use of a surface measuring model and combine it with a certain proportional reasoning argument, which again represents a non-given modelling step. The third answer even uses elementary fractions. The fourth answer focuses on a context-like surface characteristic of the given diagram and does not use a mathematical model, whereas in the case of the fifth answer, the student might not have modelled correctly, possibly by only considering lengths in the diagram.

Warum stellt die Grafik den Sachverhalt falsch dar?  
weil es 80.000 Haushalte mit Öl, 40.000 mit Gas heizen also muss das Bild mit dem Gas größer sein so ungefähr halb so groß wie das Bild mit dem Öl

“because 80,000 households [heat] with oil, 40,000 heat with gas, so the image with the gas must be bigger, about half as big as the image with the oil.”

Warum stellt die Grafik den Sachverhalt falsch dar?

Weil Öl das 2 fache ist und nicht das vierfache!

“because oil is two times [as much] and not four times [as much]”

Warum stellt die Grafik den Sachverhalt falsch dar?

Weil Gas die Hälfte benutzt wird und nicht nur ein Viertel.

“because gas is used the half and not only a quarter”

<sup>4</sup> Thus some results of the German simple we present here differ slightly from those presented at ICOTS 2010 in Ljubljana.

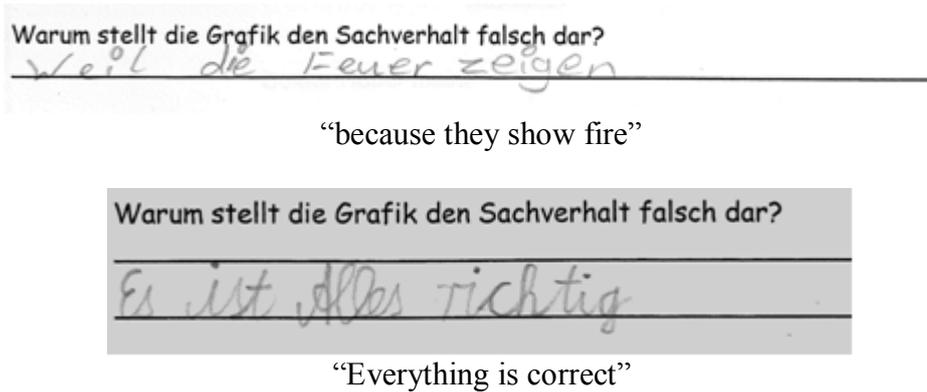


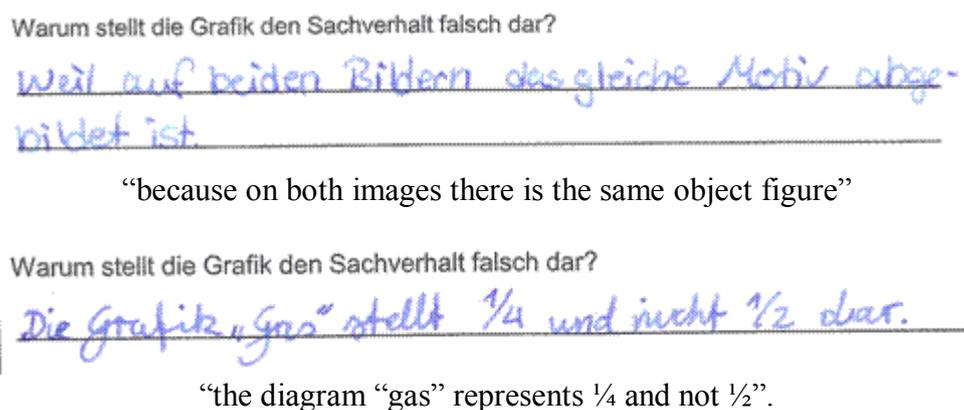
Figure 7. Examples of answers by German primary students

Taking a look at answers from German secondary students as presented in Figure 8, we can identify similar answering strategies. The first answer, for example again focuses on the surface characteristic of the signs represented in the diagram, without considering the areas in a corresponding model. This student might have preferred two different signs on the two boxes shown in the diagram. The third answer focuses even more on the flames shown in the diagram and interprets these signs as warning signs. In the case of this answer, it remains unclear whether the student had intended the statement to be a complete answer to the given question. In contrast, the second and fourth answers were coded as correct, as an adequate use of a model could be asserted in these cases. The second answer however does not clarify from which quantity the “gas diagram” represents  $\frac{1}{2}$  or  $\frac{1}{4}$ . But the answer strongly suggests that the “half” as derived from the data in the headline does not fit to the quarter as identified from the surface comparison.

A similar comparison seems to have been made in the case of the fourth answer: Again, even though intermediary argumentation steps are not made explicit, we may derive from this answer, that the sizes of the squares in the diagram have been compared to the given values in a proportional reasoning model.

The examples in Figures 7 and 8 thus indicate that the answers of the primary and secondary students were not extremely different in their quality.

Considering a problem at the highest level of competency, Figure 9 displays two answers by Colombian students. The answers show that these students were able to use the given statements about mean and deviations in a corresponding model and use it for generating data which matches to the given model-bound statements. Both answers show an adequate way of dealing with variation in statistical models and hence were coded as correct answers.



Warum stellt die Grafik den Sachverhalt falsch dar?

Man kann sich halt sehr stark verletzen und man könnte auch daran sterben. Man sollte ganz arg aufpassen.

“One may be heavily injured and one might also die from it. One should be extremely careful”

Warum stellt die Grafik den Sachverhalt falsch dar?

Weil das Gaszeichen halb so groß wie das das Öl sein sollte

“because the gas sign should be half as large as the one of the oil”

Figure 8. Examples of answers by German secondary students to the question shown in Figure 4 (level of competency III).

In contrast, the answers by German students presented in Figure 10 provide insight into potential difficulties of the learners. For the first answer, two alternative interpretations are possible: Either the student fails in modelling the mathematical meaning of the axes and expresses in his graph a high frequency of fish with a size of at most 4 cm?, whereas this frequency seems to decrease for higher values. However, this interpretation does also not fit to the given average of 30 cm. An alternative interpretation of this answer would be that the student has attempted to indicate the lengths of fish by drawing a continuous line. However in the case of this interpretation, the variability in the values appears not to have been taken into account, as all “oversized” fish appear first, before the small-sized fish.

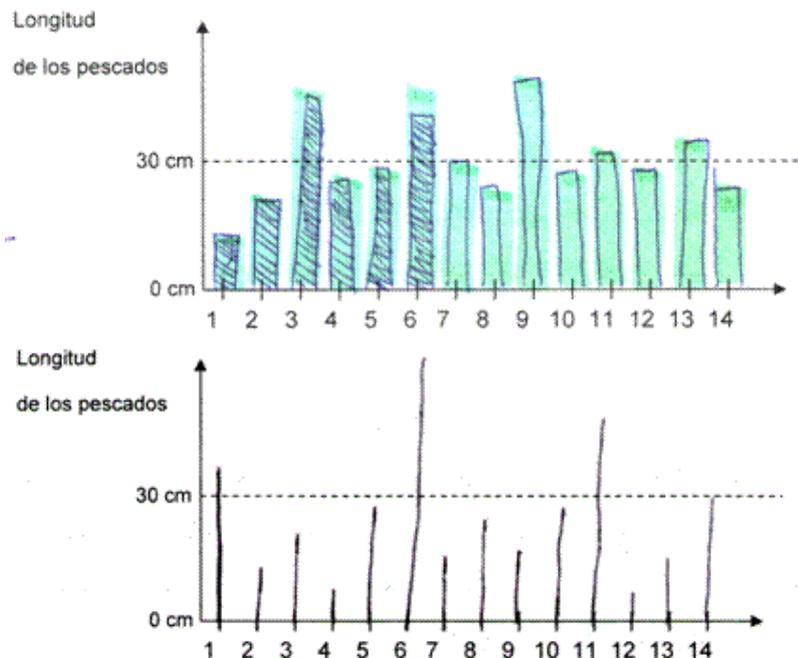


Figure 9. Examples of Colombian secondary students answers to the level V question presented in Figure 5

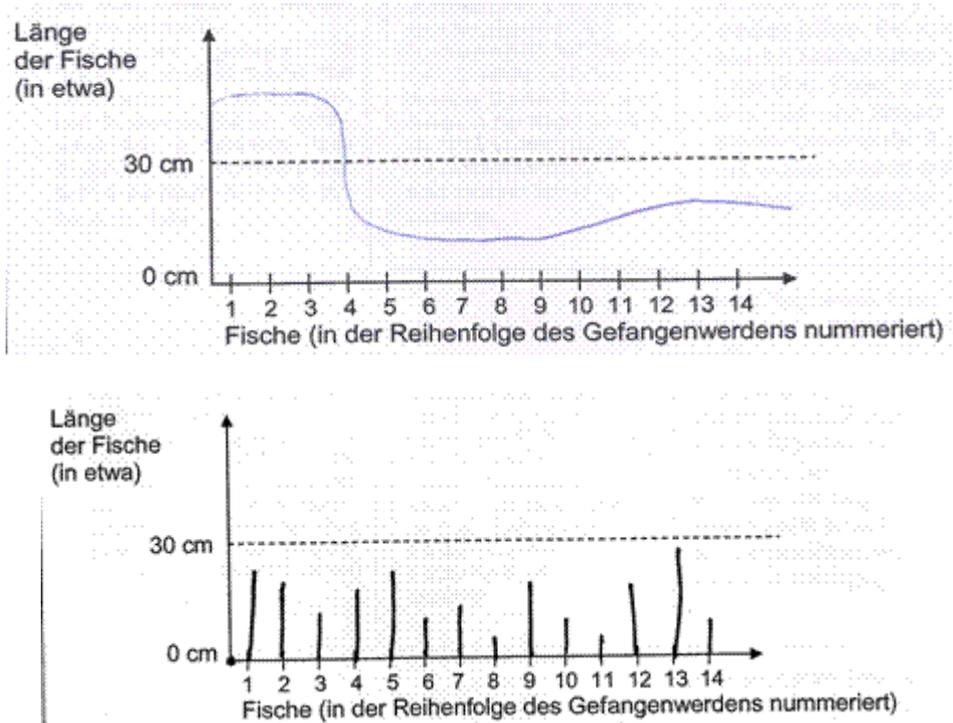


Figure 10. Two examples of German secondary students answers to the level V question presented in Figure 5

The second answer in Fig. 10 may be seen as showing awareness of statistical variation, however, the given “average of 30 cm” is obviously not reflected in the model that had led to the data in the diagram. These examples can also provide insight into the level of requirement of the problems used in the test instrument, as well as exemplify the coding process of the results. On this base, we will now consider the results from the test from a quantitative point of view.

The first and second research questions focused on the empirical examination of the hierarchical structure of the competency model and on insights related to comparisons between Colombian and German students. Figures 11 and 12 give an overview of some relevant results.

Figure 11 shows the competency scores and mean relative scores on the levels of competency for the secondary students. We observe that for the case of the secondary students the hierarchical model was consistent, as the relative solution rates decrease with increasing levels of competency. The overall scores of both groups practically coincide.

Considering the average score profiles on the different levels of competency, the Colombian secondary students performed somewhat better on the more demanding levels of competency 5 and 3, in which dealing with statistical variation plays a key role. In contrast, they reached lower scores than the German students on the more basic competency levels 1 and 2, as well as on level IV. For a deeper insight into this specific profile, we considered in an exploratory approach two different sub-samples of the Colombian secondary students, namely their school career type. As the assignment to these career types had been done recently before the time the test had been administered to the students, potential differences would be rather to be attributed to selection effects.

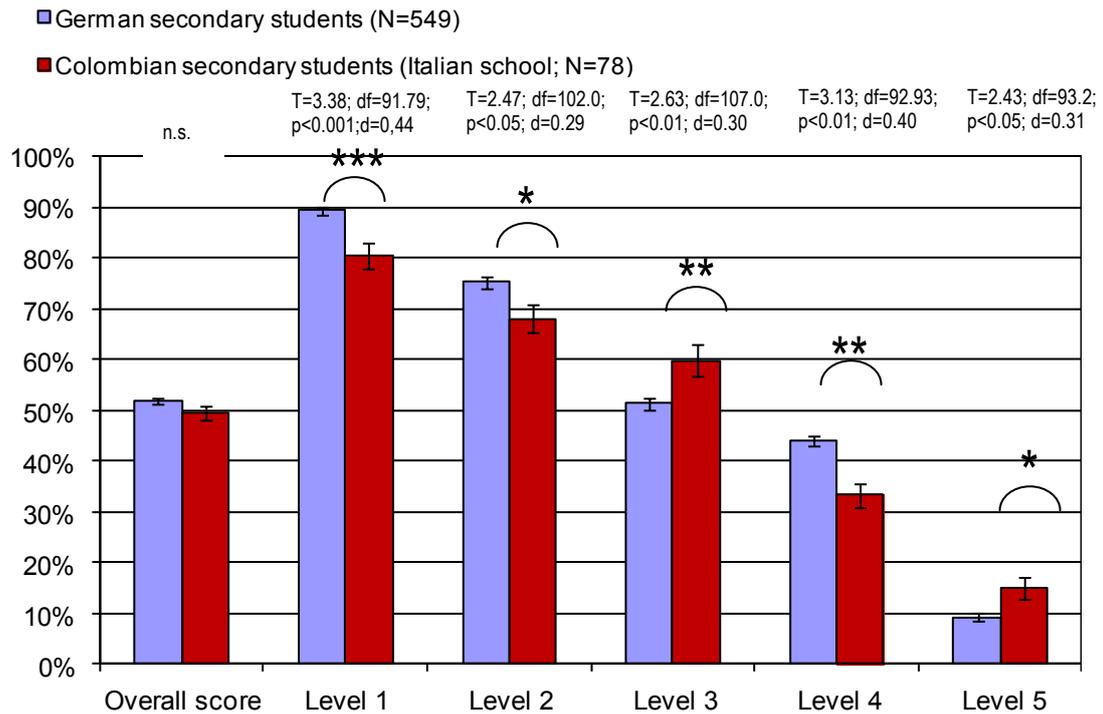


Figure 11. Competency values (overall score and levels of competency) of secondary students (means and their standard errors)

Indeed, Figure 12 shows differences between the two groups of Colombian secondary students. However, these differences are rather due to the lower levels of competency.

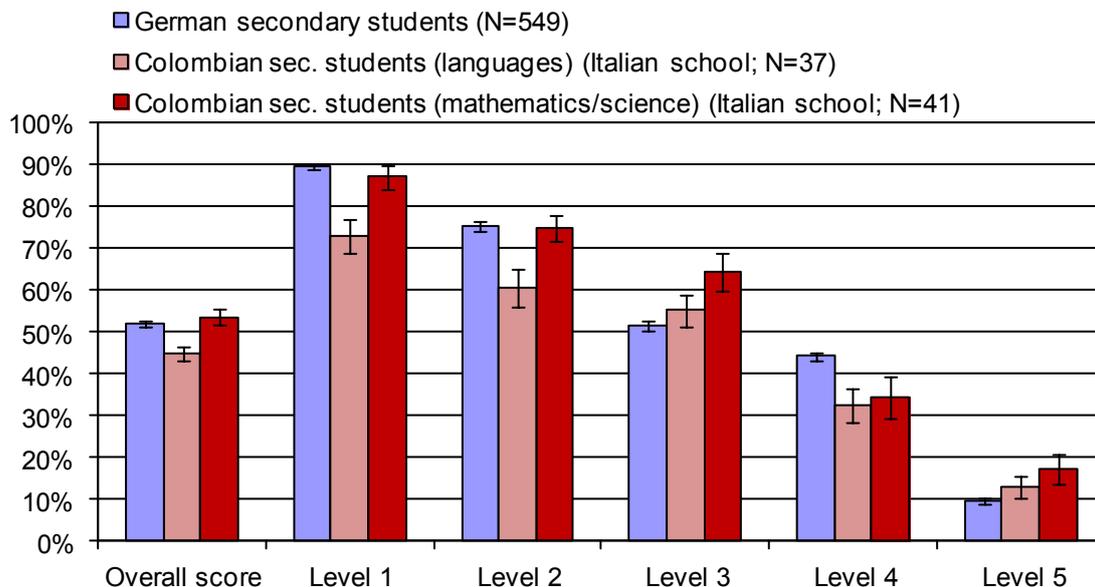


Figure 12. Competency values (overall score and levels of competency) for different career types of the Colombian secondary students (means and their standard errors)

The third and fourth research questions concentrate on the students from primary schools. Figure 13 displays the average relative overall competency score of the

primary students and the solution frequencies for problems on the different levels of competency.

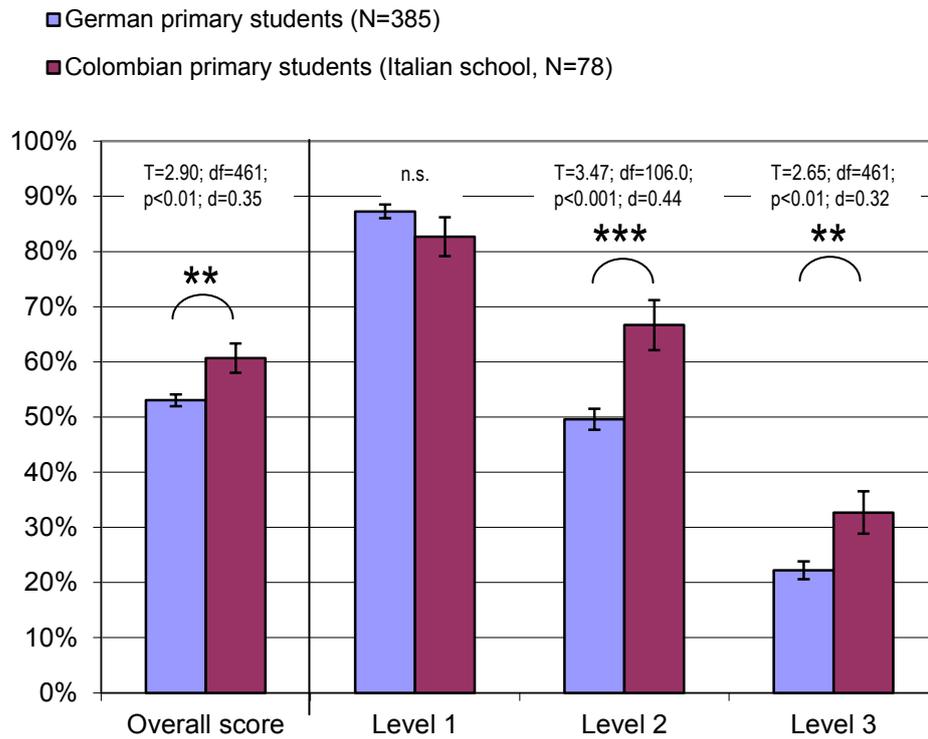


Figure 13. Competency values (overall score and levels of competency) of primary students (means and their standard errors)

Both for German and Colombian primary students, the decreasing solution frequencies with increasing levels of competency are consistent with the hierarchical structure of the competency model, in this case reduced to the levels of competency I, II, and III.

The findings suggest that the Colombian primary students performed significantly better than the German primary students, visible from the data on the left hand side in Figure 13. The effect size value indicates that this difference corresponds to a small effect. Looking at the solution rates on the levels of competency, the data show that this effect is obviously due to higher solution rates for the levels of competency II and III. The effect sizes show that these are small and medium effects.

## 7. Discussion and conclusions

The study reported here has two novel components with respect to previous work of the authors on competency levels in statistical literacy: it examines elementary school performance and includes an intercultural comparison. This intercultural study is clearly exploratory since both in Colombia and in Germany the samples were not representative. Yet the results are interesting and encourage further, wider studies.

A first important outcome of this intercultural study is that the competency levels used in the analysis are valid not just in Germany but also in a country with a completely different historical background, as far as education is concerned. Thus, as regards the first research question, the data confirm the validity of the hierarchical

structure that characterizes the competency model. Within each sub-sample, the relative scores decrease with increasing competency level.

It is relevant to note that the test given to fourth graders in Germany was performed before they were selected for the later school types (mentioned in Section 2). In contrast, the fourth graders in Colombia all belonged to one same school, which definitely presents certain homogeneity, as far as the emphasis on cultural activities is concerned. This could be an explanation why the Colombian fourth graders performed better than their German counterparts.

In secondary school the German students were more homogeneous than their Colombian counterparts, because they all belonged to the secondary school technical track (Realschule in Germany). In contrast, the Colombian students had already taken their decision, as to whether they wanted to continue on a more literary, linguistic track (Linguistic Lyceum) or a mathematical, scientific track (Scientific Lyceum). As our graph on the performance of secondary school exhibits, the Colombian sample was accordingly divided in two groups. As one can see, those who had chosen the Linguistic track performed worse than those who had chosen the scientific track.

The Colombian secondary students performed better in the fifth competency level. For instance, as was shown in the discussion, the task concerning the length of fishes was better solved by the Colombian students than by the German ones. Whether this is connected with a better reading competency of the Colombian students or with a more thorough treatment of graphs in Cartesian coordinate systems can only be the object of speculations.

The German secondary school students had had a minimal exposition to elementary descriptive statistics, while the Colombians had had none. The performance on the whole test tends to show that the statistical knowledge of the German Students was not necessarily an advantage for them.

To conclude we add some suggestions for further research topics. These may include further exploration, deepening research, in the form of a qualitative analysis of interview data dealing with questions such as:

- Are students capable of dealing with specific difficulties related to statistical variation?
- Are there other factors in prior knowledge which could play a role for the understanding of the problems?
- Broadening the analysis of other target groups including e.g. their types of school both in Germany and Colombia.

Probably, in a more general sense, in both countries, there is the potential of further improving the students' competencies related to the understanding of statistical information. Cross-cultural studies could investigate the impact of corresponding learning environments in order to contribute to the further development of mathematics instruction already at the primary level, a research goal that can clearly profit from the development of test instruments, which are not bound to a specific school culture.

All in all, a "global" tendency related to the existence of major hierarchical levels of competency emerges from our results, which are essentially stable across cultures and age groups. However, the findings suggest also a "local" aspect that consists in hints about where differences related to (school) cultures could be observed. Especially

for this second component, deepening this research is needed. Both components can help to enter the next stage of this research: exploring and evaluating ways of fostering the understanding of statistical information; identifying, describing and tackling the gap between the competencies of students today and what we may consider as a “statistically literate citizen”. As is well accepted today, statistical literacy is a goal that has a strong impact far beyond research in mathematics education.

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## **Competencias en la comprensión de la información estadística en educación primaria y secundaria: Un estudio empírico inter-cultural con estudiantes alemanes y colombianos.**

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Las competencias de comprensión de la información estadística son necesarias para la participación social de los ciudadanos y han de promoverse en la enseñanza. Por tanto, la evaluación de dichas competencias puede ser un paso para el diseño de actividades de aprendizaje que contribuyan a este fin.

Este artículo está dedicado a investigar la validez de un modelo de competencia jerárquico desarrollado previamente y utilizado con estudiantes de secundaria y que define cinco niveles, con base al uso de representaciones y modelos en contexto estadístico. Además se realiza un estudio comparado de los niveles correspondientes de competencia alcanzados descritos en dicho modelo y alcanzado por estudiantes de primaria y secundaria en Alemania y en Colombia. El estudio que aquí presentamos tiene dos componentes novedosas: analiza la validez del modelo en primaria y es intercultural.

Una amplia muestra de estudiantes alemanes (385 de 4° de Educación Primaria y 549 de Educación Secundaria (9° curso) y otra algo más reducida en Colombia (78 alumnos en cada grupo de edad y estudios similares) completaron un instrumento de evaluación. Dos versiones del mismo fueron utilizadas. Los estudiantes de secundaria completaron el cuestionario más extenso donde se consideran los cinco niveles de competencia, mientras que los de primaria utilizaron uno más reducido en que se consideran sólo tres niveles.

Los resultados confirman el modelo de competencia jerárquico ya establecido en trabajos anteriores y además sugieren perfiles específicos de subgrupos especiales de los alumnos analizados. El estudio intercultural es meramente exploratorio pues las muestras incluidas en Colombia y Alemania no eran aleatorias. Los resultados apuntan a nuevas preguntas y motivan otras investigaciones interculturales en un futuro.