Working as mathematics teacher educators at the meta-level (to the focus of the teachers on developing their teaching)

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Abstract

The professional learning of the authors, three mathematics teacher educators, is illustrated in relation to: 1) differences between being a mathematics teacher and being a mathematics teacher educator, 2) the way that novices and experts can learn in the same way through dwelling in the detail of experiences to allow new awarenesses to arise linked to new actions. The theoretical perspectives that inform the discussion are enactivism, meta-communication and relentless consistency. The practices of the three mathematics teacher educators in responding to discussions from their perceptions are at a meta-level to the pre-service teachers and support them in meta-commenting about the process of learning to the children in their classrooms. The one-year postgraduate course has served its community of schools for around 30 years in this style with relentless consistency of practices that serve creativity.

Keywords. Professional learning; teaching mathematics; teaching teachers of mathematics; awareness; meta-communication.

Trabajando como formadores de profesores de matemáticas en un meta-nivel (centrando a los profesores sobre el desarrollo de su enseñanza)

Resumen

El aprendizaje profesional de los autores, tres formadores de profesores de matemáticas, se ilustra en relación a lo que significa: 1) ser profesor de matemáticas y ser formador de profesores de matemáticas, y 2) la forma en la que noveles y expertos pueden aprender de la misma manera viviendo en detalle experiencias que permitan desarrollar una nueva consciencia vinculada a nuevas acciones. Las perspectivas teóricas desde las que se realiza la discusión son el enactivismo, la meta-comunicación (comunicación sobre la comunicación) y ser reiteradamente consistente. Las prácticas de los tres formadores de profesores de matemáticas se caracterizan porque sus respuestas a las interacciones en el aula, desde lo que ellos perciben, están en un meta-nivel para los estudiantes para profesor mediante comentarios de segundo nivel sobre el aprendizaje de los niños en sus aulas. Se describen un curso de pos-graduación de un año de duración impartido durante 30 años e implementado de manera reiterada siguiendo estos principios (en relación a las prácticas que fomentan la creatividad).

Palabras clave: Aprendizaje profesional; enseñanza de las matemáticas; enseñando a los profesores de matemáticas; ser conscientes; meta-comunicación.

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Trabalhando como formador de professores em um meta-nível (de modo a que o professor se foque em desenvolver a sua prática)

Resumo

O desenvolvimento professional dos autores, três educadores matemáticos, é ilustrado com relação a dois aspetos: 1) diferenças entre ser professor de matemática e ser formador de professores de matemática, 2) a forma como formadores em uma etapa inicial e formadores mais experientes podem aprender, de uma mesma forma, através de um hábito de detalhar as experiências de modo a permitir que uma nova sensibilidade possa surgir relacionada com novas ações. As perspectivas teóricas que informam a discussão são a enatividade, meta-comunicação e relentless. As práticas dos três formadores de professores de matemática ao responderem às discussões das suas percepções encontram-se a um meta-nível para os futuros professores e servem de suporte para pensarem sobre, e comentarem, o processo de aprendizagem dos alunos em suas salas de aula. A pós-graduação de um ano tem servido, desta forma, a comunidade de professores nos últimos 30 anos com uma consitência nas práticas que tem promovido a criatividade.

Palavras chave: Aprendizagem professional; ensino de matemática; formação de professores de matemática; sensitividade; meta-comunicação.

Travailler comme formateurs d'enseignants de mathématiques au niveau méta (pour porter l'attention des enseignants sur le développement de leur enseignement)

Résumé

Le développement professionnel des auteurs, trois professeurs d'enseignement des mathématiques, est illustré en lien avec: 1) les différences entre être enseignant de mathématiques et être formateur d'enseignant de mathématiques, 2) la façon dont novices et experts peuvent apprendre de la même manière, en s'attardant aux détails des leurs expériences afin de faire de nouvelles prises de conscience menant à de nouvelles actions. Les perspectives théoriques qui orientent la discussion sont l'énactivisme, la méta-communication et la cohérence incessante (relentless consistency). Les pratiques des trois formateurs d'enseignants en mathématiques en réponse aux discussions de leurs perceptions sont au niveau méta, afin d'aider les futurs enseignants à formuler des méta-commentaires à propos des processus d'apprentissage des enfants dans leurs classes. Ce cours au supérieur d'une durée d'un an est offert depuis environ 30 ans dans cette approche, une pratique de constance incessante au service de la créativité.

Mots clés: Développement professionnel; enseignement des mathématiques; formation des enseignants de mathématiques; conscience; méta-communication.

1. Background

The three authors of this paper have spent a substantial amount of time in their careers teaching mathematics in schools to secondary school children (from 11-18 years old). They have all taken the role that, in England, is usually termed Head of the Faculty or Department of Mathematics and all have mathematics degrees. At some point, they were appointable to academic posts at the University of Bristol, a leading UK and world university, to work, as part of their teaching commitments, with a one-year postgraduate course leading to qualified teacher status (Postgraduate Certificate of Education, PGCE). For Laurinda, this happened around 1990, for Alf, 2010 and for Tracy 2016. Tracy's post became available when Laurinda stepped down from PGCE to begin a three-year process of flexible retirement. This paper explores the theoretical perspectives and methods behind the Bristol PGCE course and the differences between teaching mathematics and teaching teachers of mathematics. What awarenesses are needed in the move from being a teacher of mathematics to being a mathematics teacher educator within this context? What theoretical perspectives support us in our development as mathematics teacher educators? How do we work individually, collaboratively and through the structures of the PGCE course?

Research on becoming a mathematics teacher educator is relatively uncommon and only a few studies on mathematics teacher educator learning exist. It is, however, an area with growing interest (see *e.g.*, Nicol, 1997; Tzur, 2001; Zaslavsky and Leikin, 2004; Even, 2005). According to Jaworski (2008), "teacher educators as researchers take mainly an outsider position in reporting their research; only a few reflect the insider position of teacher educator learning and its impact on their practice" (p. 7). Volume 4 of The International Handbook of Mathematics Teacher Education: The mathematics teacher educator as a developing professional (Jaworski and Wood (eds.), 2008) is one response to this gap in research and it is the second section of this volume, "Reflection on developing as a mathematics teacher educator", where the focus is overtly on "the mathematics teacher educator as an insider researcher developing practice through research in and on practice" (Jaworski, 2008, p. 7), which fits most closely with the subject matter of this article.

The paper begins by introducing the three authors in a mathematics teaching context where our length of experience is similar. We are considered to be expert mathematics teachers. The theoretical ideas underpinning our research and the Bristol PGCE course are then discussed, followed by a more extended piece of writing from each author to explore how the theoretical ideas fit with our developing practice as mathematics teacher educators. These perspectives, from a relatively novice teacher educator through to one with nearly thirty years' experience, are then discussed before conclusions.

2. The context of mathematics teaching and learning

In this section, the focus is three pieces of writing that introduce us as teachers of mathematics. How do we each do this?

2.1. Tracy: Matrices and transformations

I started teaching mathematics in secondary classrooms in 2002. The school where I began teaching as a newly qualified teacher was recognised as being innovative in terms of the approach to the curriculum, with year 7 (11-12 years old) and year 8 (12-13 years old) taught in mixed prior attainment groups through a series of what were called "common tasks". These may be described as projects or rich tasks that students would work on over a series of weeks.

One such project was known to teachers in the department as 'Matrices and transformations' I would always begin by displaying the same six-sided shape (Brown, 1991, see Figure 1). I would then ask, "What do you see?" to which students would usually respond in unison, "A church!" I would go on to reveal that we would be transforming the church using what we call a matrix (sometimes this would involve some conversation about the film which included this word in its title!), and that we would start by using the matrix $\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$.

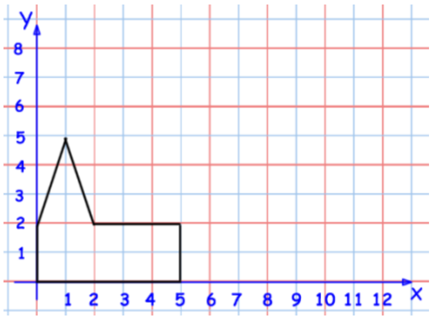

Figure 1. "A church!"

I would label each vertex of the shape with a letter alongside the coordinate of that vertex, mentioning that for this particular project we would be writing coordinates vertically. I would then ask students to watch carefully as I demonstrated the following process for multiplying the matrix by the uppermost vertex of the shape:

$$\begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} (2 \times 1) + (1 \times 5) \\ (0 \times 1) + (1 \times 5) \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$$

We would then plot the point $\begin{pmatrix} 7 \\ 5 \end{pmatrix}$ on the grid and students, when they were able to do so independently, would apply the same matrix multiplication to the other five vertices and plot the new points to form a newly transformed shape (see Figure 2).

Figure 2. Transformed church

Some discussion of the transformation itself would follow. This discussion was an opportunity to introduce mathematical terminology as well as allow students to begin conjecturing about matrices and corresponding transformations or vice versa. There was a challenge for the students over the coming weeks of, "given any 2×2 matrix, can you predict the transformation without doing the calculations?" Matrices as a topic did not, and still does not, feature on the Key Stage 3 (11-14 years old) or Key Stage 4 (14-16 years old) programme of study in England. However, I saw matrices as a meaningful context through which children could explore transformations at the same time as gaining practice with syllabus items such as plotting coordinates and drawing shapes.

Through working with students on common tasks, I was able to establish a culture with each class where an overall aim of the year was linked to "becoming a mathematician". Over many years of teaching the same tasks, I became attuned to hearing comments and observing actions linked to this aim. A powerful tool in this culture-building was the commentary that I developed alongside the doing of the mathematics. If I observed a student systematically changing the elements of the 2×2 matrix in order to test a conjecture, I would be likely to make a meta-comment along the lines of, "Great, this is a really organised approach to testing this conjecture, what do you think will happen next?" For students being less systematic in their approach, I might comment, “One thing mathematicians do is only change one variable at a time and try to understand this before changing a different variable”. Through this type of meta-commentary, it became apparent that students were motivated, asking their own questions and working on their own conjectures, to follow their own lines of inquiry as well as share ideas with peers in what would feel like a joint venture toward a common challenge.

2.2. Alf: Pick’s theorem

I have collaborated for a number of years with a charity (https://5x5x5creativity.org.uk) who place artists in schools to run projects, drawing inspiration from the practice of the Reggio Emilio pre-schools (Rinaldi, 2006). I have acted as a mathematician/artist in several primary schools. In the story that follows, a school invited me to instigate work in a classroom that was being given over to be a ‘house of imagination’ for the students. The story is reconstructed from notes made at the time. The walls were covered in plain paper for the students to write on and I had a projector at the front of the room, displaying a square dot grid.

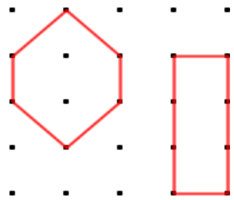

Figure 3. Two 8-dot shapes

I introduced the students to the room (not their usual classroom) commenting, “In this space you are invited to engage in “becoming a mathematician””, which I explained as meaning they ask questions, look for patterns and use their imagination. I then drew on the board two shapes (see Figure 3) and said, “These are both 8-dot shapes. Someone come and draw me another, different, 8-dot shape.”

Students came to the board and, without comment, I indicated if the shape was 8-dot or not. The distinction I needed the students to make, in this closed section of the task, was that shapes are labelled by adding up the number of dots inside and on the outside (perimeter). I also used this closed phase of the activity to set up the structure that, whenever students draw a shape, they need to write next to it, I (for the number of dots ‘inside’), O (for the number of dots ‘outside’) and A (for the area of the shape).

After several shapes had been drawn on the board that I classified as “8-dot shapes”, I invited the class to look at what had been drawn and comment on patterns or similarities and differences. One student commented that if the number of dots inside is zero (I = 0) then the area was three (A = 3), for the shapes on the board. I wrote this down as “Abi’s conjecture: with 8-dot shapes, if I = 0, then A = 3”. I collected all comments and wrote them on the board. I invited students to plan what they would like to work on within this problem. Some students wanted to try and find 8-dot shapes with more dots inside (I prompted them to try and find the biggest number of dots inside). Some wanted to try shapes with more dots and I first constrained them to stick with 8-dot shapes and see what patterns they could notice so that they had some predictions to test out before attempting other numbers. If they were not sure, they were directed to test out Abi’s conjecture. The students then broke off and work continued on tables, with patterns, questions, conjectures and any tables of results written up on the walls.

In reflecting on this lesson start, I introduced several layers of meta-communication. The broadest and most abstract is around an overall purpose for the work, linked to the idea of "becoming a mathematician". The next level down is a set of words around the mathematical processes students are invited to engage with, in particular, that this activity will be driven by the students' "conjectures", i.e., the things they notice that can be turned into predictions. These two layers of meta-communication are independent of the specifics of the task offered to the class. There is then a further set of meta-communications about this particular task, for example, that students must always write down I =, O =, A = for each shape they draw and that they must stick to 8-dot shapes initially. These communications from me about the work the students are about to do, are, from my experience working with this task, important in terms of making it likely students will generate patterns that can be noticed.

2.3. Laurinda: How many squares on a chessboard? (adapted from Brown, Reid & Zack, 1998, p. 50)

Starting to teach, I did not notice the detail of what was happening because I was too busy responding to what the children brought up. The more I used a problem, the more I was aware of all sorts of strands and possibilities, being amazed by original insights and extensions. I was able to place my attention in the learning of the students rather than in the complexities of the problem. Instead of looking for the stimulus of a new problem, dealing with the complexity allowed use of my experiences with the problem, using awarenesses that made interventions and conversations more and more absorbing.

When I taught mathematics to a new group of 11-year-olds, I always used the problem 'How many squares on a chessboard?' When I was the Head of Department, new teachers would sometimes ask, "Doesn't it get boring using the same problem again and again?" I used to say that this was my security. I liked the way the problem could be used to introduce the children to my way of working, which was based on using their ideas.

At the start of the year, the mathematics teachers who worked with the 11 year olds met and talked about what they were going to do with their classes. New teachers to the school would be a part of this group so that they could gain a sense of how the rest of us worked. One year a new teacher joined us. Later, he said that when listening to the conversation he had thought that we were quite strange and working in a way that he had neither experienced nor been trained for. Once he had worked on the squares on a chessboard for himself, he decided to see what would happen with his class. Another thing that helped him make this decision was the discussions between staff in which experiences with the problem had been described, suggesting that the beginnings of the interaction with the class were somewhat predictable.

He took in a chessboard because, he said, that made him feel more secure, although some of the rest of us got children to describe one first. When he asked, "How many squares?", what happened was what he had been led to expect (~ student; - teacher):

~ Sixty-four

- Other suggestions? [Thinking silence]

~ Sixty-five

- Why?

~ The one itself.

~ Oh, lots!

Now there was a problem that they could work on answering together, through looking at simpler cases or seeing the general in the particular 8x8 square in small groups before sharing ideas. He reported that he had been nervous until “sixty-five” came and then he thought, “They have been here before. They know what they are doing”, and relaxed. He was able to learn the problem actively through the students and compare his own attempts with theirs given the support of the other teachers’ experiences.

My role as a teacher seemed to be to respond not through giving answers but by asking questions that could lead to exploration of deeper ideas in mathematics. For instance, “There’re two hundred squares on a chessboard, Miss”, might get a reply, “How do you know you’ve got them all?” Ideas of proof, generality and algebraic ideas seemed always to be around. My motivation is related to working with the children to support their doing of mathematics. It is not the problem itself that is the important focus, rather it is the teaching and learning of mathematics. For new mathematics teachers working within a culture of experience with problem solving, joining in discussions sharing experiences of using a problem can allow them to notice and respond in their classrooms even when it is a new problem for them. It seems important in this context that there has been some work done on the mathematics first.

2.4. Discussion of the contexts of mathematics teaching and learning

The pieces of writing about teaching and learning mathematics reveal some patterns and differences. First, experience is important for the teachers working with a problem that they have used repeatedly: Laurinda “always” uses the chessboard problem with a new group of 11 year olds; Alf’s communications to students about the work are from his experiences of using the task; Tracy “always” begins her task using “the same six-sided shape.” Another pattern is the fact that learning is from experienced others. Tracy uses a problem from her departmental scheme of work that Laurinda had used and makes it her own while recognising “A church!” Laurinda describes how a new teacher commits to using a task through working with more experienced others and recognises “sixty-five”. Alf is the experienced teacher-in-residence, with awareness from experience that introducing the notation I, O, A is useful. Regarding meta-communication, Alf describes several layers of meta-commenting, that is, talking about the students’ work, e.g., “becoming a mathematician.” Tracy also explicitly mentions meta-communication, e.g., mentioning, “becoming a mathematician” as a purpose for the year for her students. Laurinda talks about responding with, “How do you know you’ve got them all?” leading to ideas of proof, which feels like a comment that would support “becoming a mathematician”. Moreover, all three teachers work with what the children bring to the situation, however structured the start of the activity, commenting on patterns observed.

In the next section, some theoretical background to these observations will be given, followed by the application of the ideas to the design of the University of Bristol PGCE course.

3. Theoretical perspectives

We discuss three ideas linked to our theoretical perspectives arising out of the reflections on the three initial pieces of writing: meta-commenting, enactivism and relentless consistency.

3.1. Meta-commenting

Both Alf and Tracy explicitly mentioned meta-communication in their pieces of writing. This idea has become embedded in our practice as mathematics teachers and for Alf and Laurinda as mathematics teacher educators. For Tracy, being able to meta-comment as an expert teacher does not lead to the awareness of how to respond as a new mathematics teacher educator. She is researching her own practice to learn how to respond in this new situation.

Pimm (1994) described some teaching as being "constantly organized [sic] by meta-comments, namely that the utterances made by students are seen as appropriate items for comment themselves" (p. 165). In this writing, we would add that the behaviours of the students in mathematics classrooms are also appropriate items for comments, e.g., "being organised".

From their study of animal behaviour, Ruesch and Bateson (1951) introduced the term meta-communication. Described as "an entirely new order of communication" (p. 209) and defined as "communication about communication", this new order allowed them to explain some complex and paradoxical attributes of social interaction. Any instance of interpersonal communication has a "report" (p. 179) and a "command" aspect. According to Watzlawick, Beavin and Jackson (1967), the report aspect of a message conveys information whereas the command aspect concerns how the communication is to be taken and therefore to the "relationship between the communicants" (p. 33). The relationship aspect of communication is "identical with the concept of metacommunication" (p. 34). The ability to meta-communicate appropriately "is not only the condition sine qua non of successful communication, but is intimately linked with the enormous problem of awareness of self and others" (p. 34). Using meta-communication in classrooms allows the children to know how to act in the moment in relation to the teacher, their peers and the community that is being built. As mathematics teacher educators, we need to meta-comment to pre-service teachers so that they know how to act when meta-commenting to the children learning mathematics in their lessons.

3.2. Enactivism

Laurinda, with Alf and with another collaborator, David Reid, have written many papers related to enactivist ideas (*e.g.*, Brown & Coles, 2011; Reid & Mgombelo, 2011). There is not space in this paper to describe enactivist principles but Laurinda and Alf have described how novices and experts in their professional learning can use practices of "deliberate analysis". Novices do not have to behave in different ways from experts when they learn. The process is of staying with the detail of an experience without judgement or justification to allow new awarenesses to arise. The key is to locate moments of ineffective action and then attempt to locate the "intelligent awareness" that led to that action, which do not suit the teacher's aims. Locating the awareness that led to the action requires a non-judgmental dwelling in the detail, to locate the possibilities for acting differently; initially what might have been done differently at the moment in question and then, crucially, what might be done differently in the future.

Enactivism comes from a biological basis of being where we see the world through our experiences, our history of interactions with it. We do not see what is really there but see what we have come to notice as important to our survival or interest. The frog catches the fly through experience, in an embodied act that happens too quickly for there to be conscious control. For us enactivism is about seeing more and seeing differently through multiple perspectives in interaction with the environment, including the people in it and what we see is related to what we do, because what we do patterns our world.

3.3. Relentless consistency

After working with the UK government to implement the National Strategies for numeracy and literacy, Fullan applied his learning to the raising of standards in literacy and numeracy in Ontario, Canada. His learning was distilled in *Six secrets of change* (2008). The six secrets are statements related to the process of working as leaders of change rather than anything to do with the content of the change process. There is no mention to literacy or numeracy, for instance. In the fourth secret, “Learning is the work”, Fullan discusses the importance of what he calls “relentless consistency” within the system, not to dampen creativity but to allow the rethinking and redoing cycle that seems to be so important. For us, what seems important is the process of using the same task to support the shift to the teacher being able to focus on the relational meta-communication about the work the children are doing on the mathematics. In enactivist terms, we act out of our history of structural coupling with the world.

4. The design principles of the Bristol PGCE course

The course that Laurinda designed around 1995 and Tracy and Alf now work on is the Bristol mathematics PGCE course. In the UK, prospective secondary mathematics teachers will have a degree in mathematics or a mathematics-related subject and apply to a university education department for a one-year PGCE course either directly after completing their degree or later in life, after having worked in such careers as being an actuary, engineering, ICT professional or even managing a pub or tree-felling! Showing how metacommunication, enactivism and relentless consistency are built into the design of the course will support the reading of writing where each of the authors writes about their learning and research as mathematics teacher educators.

The PGCE course has times in the university and periods where the pre-service teachers teach in schools. The University tutors visit the schools where the pre-service teachers are placed. The University tutors and school mentors form a community of learners where many of the mentors themselves did the course. Both Alf and Tracy have been mentors when they were in school and Tracy did the course herself.

We interview and offer places to those students who contribute to the widest spread of age; experience; and views and applications of mathematics as possible. The multiplicity of views and the fact that we, as tutors, do not believe that there is one way of teaching mathematics lead to a learning environment where the interactions and sharing between the group of prospective teachers is central. Their task, given to them at the start of the year, is to become the teacher that is possible for them. The importance of the group interactions is often commented on as part of our end-of-year evaluations. Given that our prospective teachers already have their mathematics related degrees, we do not teach them advanced mathematics as such. We do, however, spend time in workshops where they transform their learning of mathematics to extend the range of their possible offers to their pupils through listening to and working with the ways their fellow prospective teachers have of solving mathematical problems or of presenting activities to students. These ways of working consistently provide positive learning experiences from course and inspectorial evaluations. Our role during times with the group is to orchestrate the learning environment, meta-commenting at the various levels introduced by Alf in his first piece of writing, *e.g.,* in relation to the purpose for the year, to a range of responses heard, to what has not been heard and in previous years have been.

We describe the course, for students, at a meta-level. In the timetable for the Autumn Term of the course, structures emerge. On Friday mornings, there are 'Groups', where we split the cohort into two or three tutor groups dependent on personnel. The groups have the tutor who will visit them in school and work with them in reflecting time on Friday mornings at the university. Monday mornings are workshops where we work at some mathematical activities together as a class and then develop our thinking on issues that arise. Similarly, there are patterns that emerge over the year, *e.g.*, when the prospective teachers arrive back from a period of school practice, they sit in reflecting teams of three to discuss their developing practice using the details of their experiences to distil out issues. The way the course works is through the rethinking and redoing cycles of relentless consistency.

During the group sessions on Friday, we are explicit about a way of working where they share details of their practices and listen to others to extend their range of strategies, not judge what someone else offers. Over time, the group learns to trust this process and shares more openly in learning conversations. From the details of practice arise issues such as, how do we get children sharing responses to an activity? The group then develops strategies to tackle such an issue from both their observations of other teachers in the schools and their own teaching. So, the relentless consistency of these practices does not dampen creativity but supports the prospective teachers in both seeing the strategies they use as valuable to others, whilst also seeing more and differently in that they are opened up to strategies they were not aware of that become possibilities for future action for themselves. The sharing is in relation to teaching strategies that support the children to learn effectively. This is the responsibility of the pre-service teachers and they are commenting about their children's learning, whilst the university tutors are meta-commenting on these comments. As leaders of the group, we keep repeating what matters, e.g., "no right or wrong action, just what you did and reflecting on it", and there do not seem to be many of these statements. As our student teachers learn to learn about the children in their classrooms as mathematics learners, we learn about the patterns related to becoming a teacher of mathematics. The student teachers have the task of learning to teach mathematics, however, we cannot do it for them.

During the Spring Term, the pre-service teachers are in school and in the Summer Term they identify issues that they want to work on. It is during the Summer Term that Tracy engaged the group with the matrices and transformations task above, realising that she was not so comfortable beyond meta-commenting on the mathematics. There was a space created where meta-comments on the process of becoming a teacher would be possible with more experience.

The PGCE course was designed on enactivist principles, "seeing more, seeing differently". We are working to support the pre-service teachers in extending their range of practices and to do this they have to become aware of what they are not doing. This can happen through the opportunities to work in groups with peers discussing school experiences and their perceptions of the same university experiences. These "nots" are important for the pre-service teachers' learning and for our learning as teacher educators (see Alf's writing in the following section).

5. Theoretical perspectives in practice

In this section, one piece of writing from each of us illustrates how we work on our professional development as mathematics teacher educators, using the language of our theoretical perspectives.

5.1. Tracy: Working on my awarenesses through my data (PhD related)

Having moved, almost two years ago, into a teacher-educator role, I find myself reflecting on similarities and differences between my previous mathematics classroom and the room where I work alongside a group of pre-service teachers of mathematics. In planning sessions working with pre-service teachers, a useful question for me has been, "What is the purpose of this session, beyond working on the activity itself?" I decided to work with the group on the matrices-and-transformations task. In reflecting on the matrices session with the group of pre-service teachers, one issue that arose for me was around hearing and responding. Having been attuned to hear and respond to comments in a mathematics classroom, I was able to respond as a teacher but was not quite sure how to respond as a teacher educator. The purpose of the activity was "creating a culture of inquiry" on the timetable for the pre-service teachers, so I had some sense of what the session was about other than just sharing the activity. What I was less confident with was how to respond in-the-moment and what, other than my classroom-attuned responses, I could be meta-commenting upon (Helliwell, 2017). The problem of not knowing how to respond to pre-service teachers of mathematics led to me developing a research project for my doctoral studies on becoming a mathematics teacher educator, where my focus is learning how to respond to teachers of mathematics.

I am currently working with a group of ten secondary school teachers of mathematics who come together to talk about ways of developing the mathematical reasoning of the children in their classrooms. My role in the group is to facilitate a discussion where the teachers talk about what they have been doing in their schools and classrooms related to mathematical reasoning. They share ideas and stories and learn from one another. I have worked with this group of teachers for just over a year and we have met as a group five times up to this point of writing.

I am interested in how I use verbal meta-communication when responding to teachers talking about teaching, and in the process of learning to meta-communicate in-the-moment. I am in the early stages of considering my responses. Having transcribed the second discussion of the group of mathematics teachers, my analysis so far has consisted of musing over what makes a response at a meta-level distinctive to a response that is not at a meta-level, but rather at the level of the discussion. Succinctly, is the response: a) a communication about a communication or b) in the frame of the discussion? Consider the four responses in Table 1 (from the second of the recorded discussions with the group of mathematics teachers, X denotes a teacher, T denotes myself). I have included the comment immediately prior to each response.

Table 1. *Examples of responses to mathematics teachers*

Response A	
X4	Um, yeah, from what I thought would be kind of do and review of something at quite a low level and I'd have to really go over here's how you do area, here's how you do perimeter, actually it then turned into they did it all themselves, and you know in the class you get hands up all the time, it was wasn't sir help me, it was sir look at this, look at this, look at this I did it
T	Oh, that's nice, so the difference was in hands
X4	Yeah
Response B	
X6	That does definitely happen with high-ability kids as well, I was just thinking of a time a couple of weeks ago when I was doing conversions and um… We were doing area and volume conversions, but part of the starter was just simple conversions and a kid from a top set was convinced that to get from mm to cm, you times by ten and even putting examples up he still was convinced no it was times by ten. So even though he knows there are ten mm in one cm, he still was convinced you times by ten so I don't really understand how to…
T	Well it is, isn't it, you kind of are timesing by ten, it's ten times bigger, I guess maybe that's where that's coming from

Considering each response as either a) a communication about a communication or b) in the frame of the discussion, I would suggest, on first inspection, that response A is at a meta-level and that responses B is at the discussion-level. In a recent BSRLM paper, I wrote in some detail about my reaction to response A:

> I begin by considering whether "Oh, that's nice, so the difference was in hands"… qualifies as metacommunication, or, in other words, is the utterance a communication about a communication? One difficulty here is possibly with the word *about* which needs further clarification. "Oh, that's nice" is ambiguous in that the use of "that" makes it difficult to evaluate what it is that is labelled "nice". However, the second part of the utterance, "so the difference was in hands" offers an indication as to what I was valuing in that moment, using "so" as the link would suggest the "nice" was in recognition of the previous speaker's acknowledgement of an observed difference, in this case, a different reason for hands going up. Is this communication about communication? Having made the comment myself, I do of course have an insider perspective. One awareness that I know I have is when a teacher talks about a change in their behaviour or that of their students. When this happens, I find myself wanting to highlight that a difference has been noticed and how this difference has been observed. (Helliwell, 2018, pp. 5-6)

Deep in the process of analysing my responses in this detailed way, and through the act of writing, I come to a new awareness. As a head of a mathematics department, I worked hard to change the teaching of the mathematics teachers in my department to develop the type of culture across all mathematics classrooms that I had worked so hard to develop in my own through the use of meta-commentary. As a new mathematics teacher educator, my focus has become one of self-change, so that I am better prepared to support others in making changes in themselves, through the use of a different type of meta-commentary.

5.2. Alf: Awareness of "nots"

A prospective teacher (in my tutor group) emailed me to ask if we could meet after school one day to discuss difficulties he had been experiencing at his placement school. By way of background, this teacher was in the middle of a 12-week placement at a high-achieving city school. He had previously spent a six-week placement at a rural school with much more mixed levels of attainment. At that first school, he had been judged as being at a "Pass" level by the school mentors at one of the 4 Review Points of the PGCE course.

We met soon after the email and I invited the prospective teacher to talk to me about the difficulties he was experiencing. This provoked a number of stories of incidents in school. I was aware of listening with a sense of what I might be able to offer. I suspect I made little comment in between the description of incidents. I have lost the details of these stories except for the one that provoked a response in me. This story concerned an incident with another mathematics teacher in the staff room and what the prospective teacher expressed as being on the receiving end of a social rudeness. With this, an awareness crystallised: in all the stories of incidents, one thing he was not talking about was the students he was teaching. I expressed this awareness with the suggestion, it was as if his concerns were all centred around his relationships with the other teachers. I advised that he place energy and attention in his relationships with the students he teaches and forget about how he thinks the other teachers are reacting to him. Soon after expressing this awareness, our meeting ended.

I had no further contact from this prospective teacher, in relation to difficulties in school. His profile (as judged by the school) improved over the next (and final) two assessment points. Towards the end of the year I asked him what, if anything he felt had changed in the second half of the course. He mentioned several factors as having made it easier for him to deal with issues and incidents in school and teach more effectively: one was meeting a friend who had some similar difficulties he had experienced; and, one was the meeting described above.

One aspect of taking a meta perspective is listening to student teachers in a way that pays attention to whether communications feel in the "right" place, and are of the "right type." What is "right" in different circumstances will be different and I am aware of having had to work to educate my intuitions (Brown & Coles, 2000; Fischbein, 1987) of what feels "right", mainly through reflection and discussion, after teaching. An example of what I mean by a "type" of communication is that, in working on video, I will insist on a phase of "reconstruction" of events before moving to any analysis (Jaworski, 1990; Coles, 2014). In order to establish these two phases of communicating, I need to be aware of when participants are not talking within the particular discussion norm (i.e. not offering the "type" of communication required) and act to make them aware of this also. In other contexts, for example on our teacher education course, there will be explicit discussion norms of teachers offering a story from their recent classroom practice, where they have to avoid any evaluation or judgment related to the incident. Again, I will act to intervene if a story begins to slip into, for example, the teacher commenting on what they thought someone else was thinking (which is unknowable). To be able to act, to intervene, and highlight communications that are not within a desired discussion norm, involves paying attention to the content but also to the type of content, or the kind of thing being said (I have written about this previously as a "heightened listening" (Coles, 2014)).

In the story above, what I notice in relation to being 'meta' is that over the course of the conversation an awareness crystallized about the "type" of communication that was taking place. Whereas in establishing a discussion norm about using video, I will act to stop a teacher offering an analysis during the reconstruction phase, in this story my action, reflecting back to the teacher what I was hearing, happened after some time. There was no discussion norm for our conversation, meeting as we were, one-to-one and not at a university or school location. As the conversation was taking place, I was aware of a feeling of discomfort. Something "did not feel right" about the "type" of communication taking place, but I was not able to locate the source of this discomfort. With the story of the staff room incident, this discomfort resolved itself into a label for the kind of communication, it was "not about the pupils."

In working with video, I can prepare myself to impose the distinction between "interpretation" and "observation" and given that these are the two types of communication I care about, I find it is now (having worked with these two ideas for twenty years) relatively easy to impose a discipline of starting with observation before moving to interpretation. In other communications with student teachers, for example in the context above, there is a much broader range of potential 'types' of talk that either do or do not feel "right". Part of my on-going work as a mathematics teacher educator is to educate myself about what I am sensitive to. I am beginning to recognise a pattern that noticing what is "not" being said, is one way of becoming attuned to the type of communication taking place.

5.3. Laurinda: Developing as an experienced mathematics teacher educator

Varela (1999, p. 5) uses the phrase "immediate coping" that has a strong resonance for me. The process of learning as a mathematics teacher educator involves being present in the moment in relation with others and being open to awarenesses as they arise. When Alf and I worked together on the PGCE course, we would often sit and talk about recent experiences and, given that we were providing commentary at the meta-level for each other, this provided a forum for us to learn. In "immediate coping" some processes can become reified, habits become habit forming, and the stories we tell ourselves of what we are doing may possibly be revisited and open up new awarenesses. I will tell one example of this.

I have a self-perception that I am a good listener. In interviewing candidates for places on the PGCE course I had a story that I told myself that I would be listening to what they were saying. However, through our process of each of us interviewing the other, there was a time when I went silent when being interviewed by Alf. Some little time later I said, "What I've been asking myself in the last couple of minutes when I went silent is well what am I listening to if I'm not listening to what he's saying, which I suspect I've always thought I was but I'm not of course." There are two discussions that we have during interviews for the PGCE course, one in relation to "Why teaching?" and the other in relation to "Why mathematics?" I usually interview in a pair, the other interviewer being a teacher from a local school who works with us. I am an experienced interviewer and would have thought that I was listening to the content of what the interviewee is saying. However, what became apparent to me in the interview with Alf was that I do not! I am listening for something else. What?

When interviewing, we are looking for people we can work with. When things are going well, the interviewee is able to talk in detail about their experiences (e.g. visiting a school) and able to shift the level of the discussion to be about their learning. One question is important in our decision-making, “What have you learned about yourself from that visit to the school?” At the start of the interview, we say that if there is something that is blocking our offer of a place we will feed that back and there will be another chance. I am aware of the variation in the possible answers, no two ever the same, but, in the majority of cases there is no issue. These are answers good enough to know someone can learn from experience:

- That’s a good question. (long pause) I learnt that I can’t just tell them. I was talking with a girl who’d called me over because she was stuck. I told her what to do and she said she did not understand. I asked her to show me how far she’d got and this worked better. She sorted out where the problem was for herself.
- I felt uncomfortable because I didn’t know what to do when one of them misbehaves. The next time it happened I decided to try to distract back into the mathematics, “Show me how to do that one?” and it worked. I know I don’t want to end up shouting.

These are answers that I have made up. But they are a distillation of my experiences. What about when there are issues? It is hard to know when to ask the question. The issues have arisen before the question is asked, one example of which is when asked to describe a lesson that they had observed when visiting a school, the interviewee talks in terms of judgements. There were bad teachers, shockingly low achievement in the students, and the children talked! We feedback that it is best not to make judgements; rather, it is important to try to focus on what they can learn from this experience. Were all the students misbehaving? It is not until the interviewee begins to describe their experiences, rather than sharing judgements that it feels like asking the question about self is a possibility. Even so, if, “No-one’s ever asked me a question like that before” followed by anger or bursting into tears from the frustration of not being able to get in touch with their learning are the responses, then no place is offered.

In some cases, the question is asked at the point where judgements are put aside along with negative emotions and I realise that I am not listening to the context of these messages, but to the process, the meta-messages. There might be the story of an individual child’s learning, told with energy, linked for me with the idea of presence. Being here, now, “no memory or desire” (Bion, 1970) and having heard this shift, I would feel able to ask the question. The bombast disappears, with the person who arrived being who they thought we might want them to be and they answer openly, sometimes crying, and what they really fear comes out. The place is offered. I am not listening to the content of what is being said but to presence. There is similarity between Alf’s interview question, provoking the awareness/learning in me, and what can happen when I am asking the question of the interviewee.

6. Reflections

A number of patterns arise for us in reading through these pieces of writing. Working at the meta-level, there do seem to be a number of types of responses that we do not want to classify here. However, meta-comments in relation to some overarching purpose that is the underpinning to the relational interactions of the group seem central. We find evidence in the “being a mathematician” backdrop to the classrooms of Tracy and Alf (meta-comments such as “being organised”); the purpose for the year on the PGCE course of becoming the teacher they can be; iii) within the mathematics, the matrices and transformations challenge of knowing what a transformation any 2x2 matrix will perform without doing the calculation.

Another important aspect, to allow the process of “seeing more, seeing differently” is to be aware of the “nots” of the other. What is a child in a classroom not able to do? What is the pre-service teacher not talking about? What is Laurinda not doing? “Nots” are powerful ways in which, if meta-commented upon, or the space opened up, in writing or talking, for reflecting on a new awareness, for professional development to happen.

Experts and novices are learning in the same way, rather than, a model where the students copy what the teacher does or the student teachers are given a model of how to teach.

There are some obvious similarities in teaching students of mathematics and pre-service mathematics teachers. There are also important differences, even when running the same task, to the meta-level aspects the communication. For children in classrooms, their focus is the mathematics, the teacher, whilst as a mathematician they may be interested in engaging with the mathematics directly, is supporting their students through comments about their work so that the students come to know what to do in the classroom. The teacher educator’s role is similar, but more complex, in that the pre-service teacher or teacher’s focus is to support their students in doing the mathematics but the teacher educator’s role is to support the teachers by commenting about their experiences in such a way that the teacher learns to comment about their students’ doing of mathematics.

There has been a relentless consistency (Fullan, 2008) down the years at the meta-level of the PGCE course so that, even though the way of teaching is not fixed for student teachers, the way of working with university tutors has passed down through the generations of people working at the School of Education and the teachers in school form a learning community.

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Working as mathematics teacher educators at the meta-level (to the focus of the teachers on developing their teaching)

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The three authors of this paper have spent a substantial amount of time in their careers teaching mathematics in schools to secondary school children. They have all taken the role that, in England, is usually termed Head of the Faculty or Department of Mathematics and all have mathematics degrees. At some point, they were appointable to academic posts at the University of Bristol, to work, as part of their teaching commitments, with a one-year post-graduate course leading to qualified teacher status (PGCE). For Laurinda, this happened around 1990, for Alf, 2010 and for Tracy 2016. Tracy's post became available when Laurinda stepped down from PGCE to begin a three-year process of flexible retirement. This paper explores the theoretical perspectives and methods behind the Bristol PGCE course and the differences between teaching mathematics and teaching teachers of mathematics. What awarenesses are needed in the move from being a teacher of mathematics to being a mathematics teacher educator within this context? What theoretical perspectives support us in our development as mathematics teacher educators? How do we work individually, collaboratively and through the structures of the PGCE course? The paper begins by introducing the three authors in a mathematics teaching context where our length of experience is similar. We are considered to be expert mathematics teachers. The theoretical ideas underpinning our research and the Bristol PGCE course are then discussed, followed by a more extended piece of writing from each author to explore how the theoretical ideas fit with our developing practice as mathematics teacher educators. These perspectives, from a relatively novice teacher educator through to one with nearly thirty years' experience, are then discussed before conclusions.