Introduction to 'Task design in mathematics education: A diversity of theoretical frameworks'

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Introduction

The design of activities, of tasks, of sequences, or generally of any kind of curricular material for teaching, constitutes a crucial part of the research work in mathematics education. As Watson and Ohtani point out (2015), *tasks* are the basis of mathematical learning, be it from a cognitive, cultural or practical perspective. To unify terminology, in this introduction I use the term *task* deliberately and widely, and also in a certain undefined manner. Each contribution will confer this term a more accurate sense throughout the current special issue.

Task design by researchers into mathematical education can serve two different, but not necessary separate purposes. Using the distinction made by Perrin-Glorian (2011), as part of the Theory of Didactical Situations between "didactic engineering for research" and "didactic engineering for developing resources and for teacher training", sometimes task design is subordinated to the researcher's interest in inquiring about aspects of mathematics teaching and learning (see, e.g., the contributions of María Trigueros and Asuman Oktaç, of Francisco Javier García, Berta Barquero, Ignasi Florensa and Marianna Bosch). On other occasions, design primarily moves towards preparing materials for classrooms (see, e.g., the contribution of Keiichi Nishimura and Chiharu Honda). A wide range lies between both extremes, as we can particularly find in the contributions of Michiel Doorman and of Susanne Prediger, where research and materials design intermingle.

Regardless of the intended purpose, Watson and Ohtani (2015) draw attention to the fact that, when making results public, emphasis is normally placed on the "product" (curricular materials or research results), while the design process tends to be *quickly glanced at* and not truly justified in-depth. The International Commission on Mathematical Instruction (ICMI) recently dedicated one of its studies to task design (Margolinas, 2013 -Proceedings-, and Watson & Ohtani, 2015 -Volume). Likewise, during the 22 Symposium of the Spanish Society of Research in Mathematics Education in 2018, a research seminar was held about this theme, in which researchers from reference centres worldwide in task design such as the Freudenthal Institute (Utrecht University) and the Shell Centre (Nottingham University) participated.

The special issue at present continues from that seminar by putting together a group of articles from researchers who work within different theoretical frames and with distinct objectives, to make the principles, purposes and methodologies used in task design explicit. The contributions respond to the following guiding questions:

- Why do we design tasks? i.e., what are our objectives as designers?
- How do we conduct the design work? i.e., what are our design principles and our methodology?
- If design is intended to mainly create lesson materials, what impact do our materials have? i.e., under what ecological conditions do or could our designs exist?

- If design is research-oriented, what is its potential? i.e., what can be known due to the tasks designed and used in our research?

The contributions of Susanne Prediger (TU Dortmund) and of Michiel Doorman (Freudenthal Institute) lie within the frame of *design research* oriented to both the generation and validation of materials, and to the building of newer theoretical knowledge of a *local* kind. Prediger particularly introduces and discusses the integration of theoretical elements oriented to the theorisation of didactic design research. Such research is exemplified in her article by means of design principles and elements devised for teaching specific mathematical contents while paying deliberate attention to the use of language. Some of these principles are, for example, the connection between various registers and representations, or the development of discourse practices to explain and justify mathematical meanings. The distinction made between the design research tradition in education and the didactics tradition that emphasises the learning content is especially interesting. Although plenty of didactic design research has been conducted to date in mathematics education, Prediger highlights the scarcity of research that responds to the demands of producing suitable language that is specific to facilitate the teaching and conceptual comprehension of specific mathematical contents. Her article contributes to the integration of task design and learning trajectories from the dual perspective of mathematical contents and the linguistic means that these contents require to be communicated in all its complexity.

The article by Michiel Doorman combines design research and the theoretical elements developed by Freudenthal and collaborators in mathematics education. Doorman interprets task design from the perspective of designing *hypothetical learning trajectories*, which intend to describe the learning objectives regarding a mathematical object, the types of tasks to be used to fulfil them and the hypotheses assumed about students' learning processes. This author considers three fundamental principles to design these trajectories: *guided reinvention*, *didactic phenomenology* and *emerging models*. Methodologically speaking, cycles of teaching experiments are used that include the conception and design phase of the hypothetical learning trajectory, the phase in which "ordinary" classes are implemented and the retrospective analysis. Design work is exemplified for the hypothetical learning trajectory case of basic calculation principles and kinematics by modelling the movement of objects. The example describes the complete research cycle by showing how, by means of this cycle, the researcher builds a *local instruction theory* for major calculus principles.

María Trigueros (Autonomous Technology Institute of Mexico) and Asuman Oktaç (Center of Research and Advanced Studies, IPN) discuss design work within the APOS (Actions, Processes, Objects, Schemas) theory. According to this theory, the *genetic decomposition* of mathematical objects plays a key role. This decomposition is seen as a hypothetical epistemological model of structure and mechanisms involved in the construction of mathematical content. The decomposition is not necessarily unique for a given concept, but guides the design process. From the methodological point of view, on the one hand, a cyclic testing refining and validating the process of the proposed genetic decomposition is carried on and, on the other, a teaching cycle is followed in an attempt to determine to what extent students have had the chance to build all structures predicted in the genetic decomposition. The authors exemplify their work for the mathematical object case of the "inverse of a transformation matrix", which enables them to reveal the intimate relation between genetic decomposition and instructional design. They conclude by emphasising the relevance of APOS for detecting and explaining students' thinking processes, their difficulties and the relations they establish between constructions of mathematical objects.

For Keiichi Nishimura (Tokyo Gakugei University) and Chiharu Honda (International School of Tokyo Gakugei University), the fundamental objective of design work is to provide the teaching system with carefully designed tasks that improve students' mathematical learning. In Japan, this work tends to be shared by researchers (university teachers) and primary and/or secondary school teachers, and tends to be disseminated through textbooks. Nishimura and Honda explain design principles that date back to the problem solving tradition in teaching mathematics originated in the U.S., which had a strong impact on Japan and helped to outline the teaching method known as structured problem solving. In methodological terms, design is done following the *class study* schema, which entails collaborative activity among teachers to pursue detailed task design by anticipating student strategies and the discourses that the teacher could prepare. In this way, a *class plan* is devised which leads to a *research class*. The discussion of its implementation, which focuses on the thinking processes put into practice by students, is a source of professional learning and helps identify improvement points in the proposed task. This task design approach is exemplified by the task design case as part of an international project, whose objective is to develop the social decision-making competence (in this case, the best location of automatic external defibrillators in an area of Tokyo).

Finally, Francisco Javier García (Universidad de Jaén), Berta Barquero (Universitat de Barcelona), Ignasi Florensa (Escola Universitària Salesiana de Sarrià) and Marianna Bosch (Universitat Ramon Llull) deal with design activities as part of the Anthropological Theory of the Didactic. Here the design work is encompassed in a broader experimental methodology in didactic engineering terms. Any design activity starts by preparing the *reference epistemological model* of the specific mathematical object. This model plays a phenomeno-technical role because it allows researchers to identify possible didactic phenomena. The authors focus on research works that have identified didactic phenomena linked to the dominant paradigm in educational institutions, called *the visit of works* in Chevallard (2013). The need to establish a new paradigm is postulated, known as the *world questioning* that allows the negative effects associated with some identified phenomena to be reduced. This new paradigm is characterised by a functional approach to mathematical knowledge given the study of crucial problematic questions that produce understanding and the starting of an investigation process. The *study and research paths* are the *tasks* that researchers design within this frame. The approach is exemplified by: the phenomenon of disarticulating the study of functional relations in secondary education and the design of the "savings plans" *path*; the phenomenon of *applicationism* in the teaching of mathematics as part of university degrees of experimental sciences, and the *path* about "population dynamics"; the phenomenon of the linear elastic model's *sliding of the raison d'être,* presented as a purpose in itself by displacing the real problems that it helps solve through a *path* about "designing a piece in a real context". The design of each *path* is preceded by an explicit construction of a reference epistemological model. The authors end with the discussion of the ecology of task types as *study and research paths* and the non-normative nature of their vision of mathematics didactics.

This special issue cannot include the vast richness of design principles, purposes and methodologies that exist in the community of mathematics education researchers. Nonetheless, it can be useful for inviting any researcher to reflect on, clarify and share theoretical elements, principles and purposes, and also the methodologies used when designing tasks. This work is crucial for continuing to advance in gaining a better understanding of our work as researchers, and for a profounder and more prolific dialogue between perspectives and theoretical frameworks.

References

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