Experiences of problem solving in whole class interactions

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Experiencias de resolución de problemas en interacciones del grupo clase

Abstract

La resolución de problemas a menudo se considera una parte esencial del aprendizaje matemático. En este artículo examinamos las interacciones de toda la clase en torno a los problemas y su resolución, tal como ocurren naturalmente en el aula de matemáticas. Por tanto, examinamos experiencias ordinarias de los estudiantes en la resolución de problemas en sus sesiones habituales de clase. Nuestro análisis muestra cómo los estudiantes participan en una gama muy limitada de acciones de resolución de problemas y que las acciones en las que sí participan son controladas por el maestro. Esto plantea implicaciones acerca de cómo los estudiantes perciben e interpretan resolver problemas en matemáticas.

Palabras clave: resolución de problemas; análisis de conversación; interacción en el aula.

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Abstract

Problem solving is often considered to be an essential part of learning mathematics. In this paper we examine the whole class interactions around problems and problem solving as they naturally occur in mathematics classrooms. Thus, we are examining students' ordinary experiences of problem solving in their everyday mathematics lessons. Our analysis shows how students' participate in a very narrow range of problem solving actions and that the actions that they do participate in are controlled by the teacher. This raises implications for what students perceive and interpret problem solving to be in mathematics.

Keywords: Problem solving; conversation analysis; classroom interaction.

1. Introduction

Thompson (1985) argues that "to learn mathematics is to learn mathematical problem solving" (p. 190). The relevance of problem solving in learning mathematics is reflected in the curricula around the world. In the UK the Cockcroft Report (1982, paragraph 249) stated that "mathematics teaching at all levels should include opportunities for problem solving" and one of the aims of the National Curriculum for 11-14 year olds states that all students "can solve problems" (English Department for Education, 2013, p. 2). The standards for mathematical practice in the Common Core in the US states that students need to make sense of problems and persevere in solving them (http://www.corestandards.org/Math/Practice). In both the UK and the US practices like reasoning and arguing are mentioned separately. In Singapore, problem solving is at the centre of the framework of school mathematics curriculum (Kaur & Toh, 2011) and treats reasoning as a key component of problem solving.

In this paper we focus on students' experiences of solving problems as they are enacted within the classroom. However, we differ from existing work on problem solving in the classroom by considering problems to be those tasks that the teachers describe as problems, and the activity of solving problems as those activities or actions that teachers and students perform during whole-class discussions around the teacher's problem, i.e. those actions and practices involved in the activity of problem solving that are public. Students are offered a variety of tasks in their mathematics lessons, only some of which can be described as problems, and only some of which are followed by

the activities associated with problem solving. By considering only those tasks described by the teachers as problems, we differ from the usual researcher's definition of a problem as a task where there is no readily available procedure for finding the solution (Hodgen, Foster, & Kuchemann, 2017, p. 14) or a task where someone "experiences a state of problematicity" (Mason, 2016, p. 263). These definitions are subjective in that for some students a task may be a problem, whilst for others in the same class it may not. This usual definition is also one that researchers use, and does not necessarily match what teachers or students consider to be problems. We are interested in students' experiences of problems and of problem solving in classrooms and we do this by considering those tasks that teachers explicitly describe and pose as problems and analysing the work that is done in the interaction that follows.

2. Literature review

2.1. The nature of problem solving

The extensive research on problem solving in mathematics argues that problem solving involves: understanding and making sense of the problem; connecting the problem with known information, ideas, procedures and strategies; using multiple representations; recognising similarities in the structure of different problems; metacognitively analysing a problem, the problem solving approach and the solution (Guberman & Leikin, 2013; Mason, Burton, & Stacey, 2010; Schoenfeld, 1985). The questions then arise as to whether students themselves are performing or experiencing these activities, and as to whether the students are aware that they are engaged in these activities, or are supported to become aware. Research that focuses on students working independently from the teacher, but possibly collaborating with other students, have detailed the strategies and approaches students use (Chan & Clarke, 2017). There is now considerable literature on mathematical problem solving inspired by the work of Pólya (1945), Mason (2010) or Schoenfeld (1985) (see, e.g., Felmer, Pehkonen, & Kilpatrick, 2016). There has also been considerable research on the design of tasks suitable for engaging students in problem solving (Stein, Grover, & Henningsen, 1996). Yet there is little research examining problem solving as it naturally occurs in classrooms. In this paper we compare and contrast two whole-class interactions around solving a problem in secondary mathematics classrooms with 13 year-old students in England.

The stance we take is that problem solving is co-constructed by teachers and students in mathematics classrooms but it is the teacher that societally validates (Kim & Roth, 2018) what is said and done. That is, what the teacher accepts and what the teacher emphasises (see, e.g., Goizueta & Planas, 2013) is what students learn to be mathematically acceptable. As Yackel and Cobb (1996, p. 461) put it, the "teacher can serve as a representative of the mathematical community in classrooms" and their validation and acceptance of student explanations or justifications influences what counts as a mathematical explanation or justification. What it means for students to solve problems can thus vary between classrooms, and consequently what students learn to be mathematical problem solving, through successful participation in interactions around the solving of problems, varies between classrooms.

Tasks and problems can offer students the opportunities to engage in a range of problem solving activities, but they do not guarantee that students will recognise or seize these opportunities. It is not enough to provide students with cognitively demanding tasks, for the teacher also needs to support and develop students' problem solving through how they interact with them. Students also need to be aware that they solving

problems or problem solving, through teachers labelling tasks as problems and activities as problem solving. Using Marton and Tsui's (2004) distinctions, it is the enacted object of learning that we consider here, not the actual or intended object of learning, by examining what teachers and students actually do when interacting around a shared problem. It is with this in mind that we now turn to the literature on teacher practices that support students' mathematical activity.

2.2. Research on mathematics classroom interaction

The research that considers mathematics classroom interaction is extensive and examines a range of issues within mathematics education. In this section we consider research that focuses on teachers' or students' practices that support the activities of doing mathematics. These include argumentation, explaining, and reasoning, for example. As teachers can play a pivotal role in orchestrating these mathematical activities and practices, we also consider some of the interactional moves that teachers can make to support these practices detailed in the literature that we draw on in the analysis below.

Mathematical practices such as explaining, reasoning, and arguing are developed through classroom interactions. As mentioned above, what counts as a mathematical explanation or justification (Yackel & Cobb, 1996) is given by what teachers, and to some extent students, treat and accept as mathematical explanations or justifications. However, both Erath (2017) and Ingram et al. (2019) show that teachers accept a wide range of contributions as explanations in mathematics classroom interactions raising the issue of what students are learning about what counts as a mathematical explanation rather than an explanation (Erath, Prediger, Quasthoff, & Heller, 2018). This distinction is made by Yackel and Cobb (1996) when they contrast social norms and sociomathematical norms.

Students' argumentation has been widely studied, focusing on how students justify the claims they make and the evidence that they draw upon. Collective argumentation specifically focuses on argumentation in interaction and has been considered by Krummheuer (2007) and Conner et al. (2014) amongst others. Collective argumentation is where teachers and students co-construct mathematical arguments which include mathematical claims and evidence to support these claims. From this interactional perspective, what counts as data or a warrant is jointly negotiated in interaction. Yackel (2002) focuses on what teachers can do to support collective argumentation, such as not evaluating students' reasons but inviting others to comment on them, or offer different reasons and shifting students' attention away from the claims being made towards the supporting evidence for the claims.

Research focusing on the actions of teachers in supporting mathematical discussions and interactions identifies a variety of actions at different levels of generality. These can include guiding principles or ground rules, such as Staples' (2007, p. 172) guiding the mathematics, establishing and monitoring a common ground, and supporting students in making contributions, to very specific patterns of interaction such as focusing or funnelling (Wood, 1988) or even specific utterances such as revoicing detailed below. For example, Stein, Engle, Smith and Hughes (2008) outline five practices for teachers to use in orchestrating whole-class discussions. Four of these describe actions during lessons: monitoring students' responses to the tasks; selecting particular students to present; purposefully sequencing the student presentations; and helping to make mathematic connections between the responses. These actions relate to a particular teaching structure where students explore the task independently from the teacher before

a whole-class discuss-and-summarize phase. Yet work on tasks can use a variety of structures, including oscillating between independent work and whole-class work. Yackel (2002) and Conner et al. (2014) both focus their descriptions of teachers' practices on those that support argumentation, including direct contributions where the teacher provides part of the argument being co-constructed such as a warrant or backing, and by asking types of questions or providing prompts, including requests for elaborations or evaluations.

However, opportunities for participation in explaining (Erath et al., 2018) and argumentation (Cramer & Knipping, 2018) are not evenly distributed across students, classrooms or schools. Many studies present examples of rich interactions that involve problem solving or collective argumentation (e.g., Whitenack & Knipping, 2002) but these examples serve to illustrate particular practices, or even to advocate a different way of teaching, and do not necessarily reveal the everyday lived experiences of students in the mathematics class. An ethnomethodological approach, such as the one taken here, focuses on these ordinary, everyday experiences. It is these experiences that establish the norms around what it means to solve problems in specific mathematics lessons.

Many of the aspects of practice discussed above are used by the teachers in the extracts we share below, yet not necessarily in a way that supports collective argumentation, or the articulation of students' reasoning or explanations. It is often the sequence of interaction within which these practices occur that establishes the norm or that influences students' behaviour, rather than the individual practices themselves. For example, Wood (1988) makes the distinction between a funnelling pattern and a focusing pattern of questioning, with a fundamental distinction being between whether the goal of the interaction is the teacher's anticipated solution or is a shared goal with the students. A funnelling pattern describes a series of teacher questions that directs students through a specific journey to a desired solution. This pattern can give the impression of learning even though it is the teacher that has done the cognitive work through the design of their questions. Focusing interactions in contrast are patterns of interaction where the teacher's questions respond specifically to the students' responses and the direction is toward a shared goal.

Revoicing (O'Connor & Michaels, 1993) is another widely discussed teacher practice that involves the repeating or rephrasing of a students' utterance in a way that allows the student to affirm or contest what is repeated or rephrased. It is this evaluation of the repeating by the student that marks this move as different from simple echoing or repeating which is very common in classroom interaction and can perform a variety of discursive actions (Lee, 2007). Revoicing that involves the teacher changing what the student said in some way that emphasises or draws attention to a particular idea, work or aspect of what the student said is called active revoicing (Eckert & Nilsson, 2017). Herbel-Eisenmann, Drake and Cirillo (2009) identify four features of a revoicing move. Firstly, they often begin with a discourse marker such as 'so', also identified by O'Connor and Michaels, which indicates that the turn is building on what has just been said. They also include personal pronouns such as 'you' to create a relationship between what the revoicing and the student who gave the original utterance. Next there is a laminating verb like 'think' or 'said' which further reinforces the connection to the student. The final feature is the one emphasised by O'Connor and Michaels which allows the student the opportunity to affirm or contest what has been revoiced.

In the analysis of the two problem solving activities we describe below we focus on the whole class interactions, in what could be called *collective problem solving*. Whilst

students may have worked on the problems independently, it is how they are discussed publicly and how the process of solving the problem is jointly negotiated by the teachers and their students that is the focus of the analysis. It is this public interaction that reinforces what the teacher treats as important, and what is emphasised and made public, and what is ignored. We acknowledge that the activities and practices that students may participate in when working individually or with their peers is likely to be different from those they participate in when interacting with the teacher (Goizueta, 2019), by focusing on whole class interactions we can study what opportunities all students have to see or participate in particular practices involved in mathematical problem solving.

3. Theoretical approach and methods

The analysis takes a conversation analysis approach (Sidnell & Stivers, 2012) grounded in ethnomethodological principles (Ingram, 2018). This means that in the analysis it is the perspective of the participants that matters, i.e., what teachers and students treat as problems and problem solving. This approach emphasises the social and interactional nature of teaching and learning mathematics and focuses on naturally occurring interactions. The lessons considered here were not specifically designed to be about problem solving, and therefore any problem solving activities arose naturally as part of the teachers' usual practice. It is an inductive approach to research focusing on sequences of utterances in naturally occurring interaction. It is also a micro approach where utterances are considered to be social actions, and consequently, we take learning mathematics to be about doing mathematics: such as explaining, justifying or defining. For this reason, it is essential to analyse students' learning of mathematics in classroom interactions at the micro level.

3.1. Data and methods

The data considered comes from a corpus of 52 lessons with 17 teachers from 8 schools. These schools were diverse in terms of levels of social deprivation, proportion of students with identified learning difficulties and proportion of students with English as an Additional Language.

The whole-class interactions from these videos were transcribed using standard Jefferson transcription (Sidnell, 2010). Teachers have been given pseudonyms beginning with T and students have been given pseudonyms beginning with S where an individual student can be identified. Where it is not clear which student is speaking we have used S to denote a student speaker, or Ss to denote multiple student speakers. A collection of cases was then built of tasks where the teacher, or a student, used the word 'problem' at some point during the introduction of the task. This building of cases omitted those situations where the teachers talked about problems in terms of a difficulty rather than a task, though analysis of these difficulties could in the future offer further insight into the classroom practices associated with problem solving. Two of the teachers, not considered in this paper, videoed two consecutive lessons based on an investigative task, which could be classified as a problem using the traditional definition. However, in neither case did the teachers use the words problem or problem solving when they introduced the tasks. Both these teachers described these lessons as 'doing something a bit different'.

This paper presents two episodes that illustrate how classroom interactions influence the problem solving experiences of students in mathematics. These two cases are typical of the handling of problems in interactions within the data but also offer an illustration of some of the differences in the ways that teachers and students engage in the problem

solving process. By focusing on teachers' instructional practices during whole-class discussions on the solving of a problem, we offer a different perspective on the nature of problem solving in the ordinary classroom, compared to studies using researchers' definitions of problems and problem solving.

4. Problem solving experiences in whole class interactions

4.1 Tyler's lesson

Tyler introduces the activity as a game but does not use the word problem at this stage to describe the activity or questions. The game involves ten paper cups, one of which has a red cross marked inside it. The cups are lined up and the students are invited to choose a cup, one at a time and without replacement. Between turns 11 and 60 the class are playing the game and the focus is on finding the cup with the red cross. In turn 60 in Extract 1, Tyler poses a question about whether the students are better off being the first ones to pick a cup, or whether the chance of finding the cross increases if you wait. The students 'guess' positions to play before the game is played again. It is not until turn 157 that the word *problem* is used and the question about when to choose a cup to have the best chance of finding the cross becomes about likelihood and probabilities.

Extract 1: Tyler introduces the problem

11	Tyler:	… um. today though I wanted to look a little bit at probability. okay. we've done some of this before and I wanted to push you on a little (.) bit (0.3) further with some of the probability work that we've done. so. first thing is it is my birthday today so I thought we'd play a game to start off with
		((transcript omitted))
60	Tyler:	… okay my next question would be (.) okay (.) is it better, (1.8) is it better to go first, (1.5) or is it better to hang on and wait. what's more likely. when are you going to be more likely to win (.) if you (.) wait (.) if you go first? if you go last? if you go somewhere in the middle. where's the best position to actually have (.) a guess do you (.) think. Joe
		((transcript omitted))
193	Tyler:	… so thinking about this (1.1) as a problem. think about this when's it most likely to choose (0.4) which one? okay? what's the probability of the first person winning. what's the probability of you getting it right straight away.

The game becomes a problem in the same turn as the students shift from thinking about winning the game to thinking about likelihood and probabilities.

Extract 2: Tyler's class work on the problem together

193	Tyler:	… so thinking about this (1.1) as a problem. think about this when's it most likely to choose (0.4) which one? okay? what's the probability of the first person winning. what's the probability of you getting it right straight away.
194	Sam:	er one in ten
195	Tyler:	one in ten. good. what is the probability of the second person winning. okay, (0.5) think about it. what do we need the first person to do.
196	Ss:	get it wrong
197	Tyler:	get it wrong. so what's the probability of the first person getting it

		wrong.
198	S:	one
199	Sam:	nine tenths
200	Tyler:	nine tenths. what's the-, an-, and we want the second person to win. so the probability of the second person winning is what.
201	S:	nine[ty ni]ne [((inaudible))]
202	S:	[eight]
203	S:	[sir] wha- what would you do if that [((inaudible))]
204	S:	what number?
205	Tyler:	it doesn't matter because we're not playing at the moment, just ((inaudible))
206	S:	oh
207	Tyler:	what's the (0.3) probability of the second person winning?
208	S:	one in nine
209	Tyler:	one in <u>ni</u>:ne. what do I get if I multiply those together.
210	S:	er nine[ty nine]
211	Sam:	[nine over ninety]
212	Tyler:	cancel it down
213		(1.4)
214	Sam:	three in thirty
215	S:	three in thirty
216	Tyler:	cancel it down again!
217	S:	one in ten
218	S:	one in ten
219	Tyler:	one in ten. ex<u>act</u>ly (.) the same (.) probability. second person has e<u>xa</u>:ctly the same chance (0.6) as the first person. the probability of the second person getting it is exactly the same. do it for the third, (0.8) we want the first person to lose. what's the probability of the second person losing. ((transcript omitted))
243	Tyler:	so I can just cancel the down straight away. so, despite what you thi:nk (0.4) it doesn't matter when you go. you still have the same (.) probability if y- if you chose before now which position to go in, you would have the same probability of winning (0.6) no matter where you go=

In turns 193-208, Tyler is breaking down the problem into separate questions that ask the students to calculate the probabilities of single events and to recognise what the events are, such as the need for the first person to get it wrong in turn 196. The sequencing of the questions asks students to calculate a single probability at a time, with student turns that do not give the required probabilities being ignored, as in turns 198, 201 and 202. With each appropriate answer Tyler repeats the answer before moving on to the next question, effectively evaluating these answers as correct by doing so (Lee, 2007) . In turn 209, the students are asked to combine the probabilities, but Tyler tells them how to do this so that the students are only required to perform the multiplication of the two fractions, nine tenths and one in nine. Tyler then explicitly asks them to cancel it down in turn 212 and again in 216, and the students respond with simpler fractions each turn. They reach the final answer of one in ten in turns 217 and 218 which Tyler repeats. It is Tyler who marks this as the end of the calculations and summarising what one in ten is the answer to. It is Tyler who makes the connection between the probability of the first person winning and the second person winning as being exactly the same chance. The interaction then continues with the calculation of the probability of the third person winning in much the same way as the interaction around the probability of the second person winning, though with some difficulties with the simplification at the end.

The work on the problem ends with turn 243. Here Tyler summarises the conclusion that each of the probabilities are the same and therefore it does not matter which position you are in for choosing a cup. Whilst Tyler draws upon his students' turns in his own turns, it is Tyler himself who has connected the three probabilities and identified that they are the same, and it is Tyler who has used this to conclude that it does not matter which position you go in.

4.2 Tim's lesson

In the second example, Tim introduces the 'problem' as being about averages, which includes the mean, mode, and median. Tim clearly states what is needed to be known in order to solve this problem, which is the question of what mark will Michelle need to get in her fifth exam in order to have an average mark of 70%.

Extract 3: Tim introduces the problem

| 2 | Tim: | I just want to look at a little problem,(5.5) I want to look at a little problem involving <u>averages</u>, just remember we are now <u>experts</u> at the mean, mode, (.) median (0.4) and if necessary the range. so I'll show you this in a second, just want you to think about this, perhaps talk to the person next to you. I'm not going to answer any questions until I think everyone's had a go at at it and then we'll, we'll um have a look on the board. ((transcript omitted)) (1.7) right have a look at this then. (3.1) if you want to do any working out you can do that in the back of your exercise books. (4.8) Michelle's parents agree to buy her a new surf board, she <u>has</u> to get an average test score of <u>seventy</u> percent or over. she's done (1.1) four exams already, that's what she got. My question is what will she need to get (0.7) in maths, at <u>least</u> what will she need to get, (1.0) if she's going to get this average score of seventy percent. you might need a calculator for this I don't know. |

Tim gives students time to work on the problem themselves, talking to each other whenever they wish. He makes reference to the work they have been doing on the mean, mode and median which are the averages they will need to solve the problem, and he makes this connection explicit. The working out that they need to do can be done in the back of their books and they may need a calculator to do some of the calculations. The problem is described as 'little' twice and will require students to 'think'.

Tim begins the whole class interaction around the problem by inviting students to explain "what they were thinking about" and then to report "possible ways of doing it". Sam is the first student to volunteer to report what he had been working on and he describes the process of what he did. This process is a calculation that involves adding all the numbers on the whiteboard and then dividing by 5. Sam does not offer any reasons for why he did this and Tim does not prompt him for these reasons. Sam has followed an algorithm similar to finding the mean although this is not explicitly mentioned by Sam or Tim in this interaction. However, there is problem with Sam's answer which Tim hints at in turn 16 where he revoices Sam's explanation adding up *the* four numbers, to adding up four numbers, and then divided by five. Sam does not change his answer. Tim then repeats the process Sam is reporting but this time inserting the numbers Sam would have added in the first stage, and Sam affirms that this is what he did.

Extract 4: Tim's class begins to work together on the problem

| 8 | Tim: | okay then folks. (0.8) can we have quiet please. (0.7) either (.) you've got this figured out, in which case (.) listen or contribute, or you |

haven't got this figured out, (0.3) in which case you need to also listen and to try and (.) work out th-, the i- the way of thinking to solve these types of problems. u:m (0.8) right I got my answer in a flash, I've got to admit it (.) okay (.) but u:m (0.3) somebody else then, can we have someone explain to the class what they were thinking about, (0.4) possible ways of doing it please. (0.7) Sam.

9	Sam:	I added them all up
10	Tim:	you added them all up
11	Sam:	yeah
12	Tim:	so you did (0.9) what that (0.6) plus that (0.5) plus that, did you add that one on as well.
12	Sam:	er:: no
13	Tim:	okay
14	Sam:	and then (0.6) I (1.5) divided that by five (0.7) to get the how much she needed (0.7) in the last (0.3) um: (.) test.
15		(1.2)
16	Tim:	so you added up the four numbers, (0.8) you added up four numbers (0.3) and then you divided by five? (1.8) is that it?
17	Sam:	yeah
18	Tim:	so seventy two plus forty three, plus eighty five plus seventy one. wha- Sam what did that add up to.
19	Sam:	two hundred and seventy one
20	Tim:	two hundred and seventy one is it.
21	Sam:	yeah
22	Tim:	so that eq- that equals two hundred and seventy one and then you did two hundred and seventy one divided by five. °Oh okay°.

This whole sequence has been about the procedure Sam used to solve the problem (as he states in turn 14). No other aspects of the problem solving process are explicitly discussed or drawn upon, including that the calculation is about the mean. Tim does not evaluate or make the part of Sam's answer he has a problem with explicit until turn 28 in Extract 5. Tim does offer an explanation for why he sees Sam's process as problematic at the end of this turn, but at this stage he does not offer Sam the opportunity to change or correct what he did. Instead he invites Steven to take the next turn – to offer another strategy. Steven reframes the question to be about finding the total that needs to be divided by five in order to achieve a mean of 70. Tim revoices Steven's process in turns 33, 35 and 37 with Steven affirming Tim's interpretation in turns 34 and 38. This extract also includes the first mention of mean in turn 33.

<u>Extract 5: Steven offers a new interpretation of the problem</u>

28	Tim:	okay. Sam added them up. okay (1.2) Sam added them up shhshh shh shh. Sam added them up, they added up to two hundred and seventy one, that is a useful bit of information (1.0) but that thing about dividing by five. that seemed to me, I don't know, a little bit nonsensical cause you've only got four numbers, dividing by five (0.6) I'm not sure. (0.4) Steven.
29	Steven:	um you need, if yo-, you can find th- like (0.3) all the numbers, the end mark, the end (.) percentage means that there's like three hundred and fifty percent altogether so if you divide by five it comes up to seventy.
30	Tim:	right hold on a sec. (0.6) three hundred and fifty percent, (0.2) er I suppose, can you add percentage together and then get three hundred and fifty per

		[cent I suppose so]okay
31	Steven:	[no what we]
32		(1.0)
33	Tim:	so you're saying that if you've got five numbers (0.4) and you want to get a mean (0.3) of seventy
34	Steven:	yes=
35	Tim:	=those five numbers must add up to (0.7) [w]hat
36	Steven:	[thr-] three hundred and fifty
37	Tim:	three hundred and fifty.
38	Steven:	ye:[h
39	Tim:	[okay] that is, does everyone understand that idea.
40	Students:	yeah
41	Tim:	so then if this, (1.2) hold on, you've got some mystery number (0.4) and when you divide by five you need to get seventy percent. Steven's saying that mystery number on the top (0.3) has got to be (0.4) three hundred and fifty. [yeah (.) oka:y]
42	Steven:	[because seventy times five equals]
43		(0.6)
44	Tim:	so then, (1.1) we know this. you know that those (.) four add up to two hundred and seventy one, (0.4) so then I suppose what you could do (.) is say that two hundred and seventy one plus the maths mark, ((writing on the board at same time)) well has got to equal three hundred and fifty doesn't it. (1.2) does that make sense? [yeah Sean.]
45	Students:	[yeah]

In Extract 5, the focus is on the reinterpretation of the problem alongside the calculations needed. Steven begins by stating what he wants to find out, 'the end mark' which he then gives without explicitly explaining how he reached a total of 350%. Tim gives an explanation as he revoices Steven's responses in turn 33, however, this explanation is at the general level and it is not explicit about which five numbers must add up to 350 and this calculation is not possible because of the missing number. Steven gives the actual calculation needed to get 350 in turn 42 but overlapping with the teacher. Tim does not use this explanation in his summary in turn 44 which rephrases Steven's reframing of the question from turn 29, drawing upon the value of 271 Sam found in Extract 4. Tim emphasises the role of using what they know at this point, which is the value of 271 that Sam found by adding up the four marks written on the whiteboard.

Extract 6 then continues with Sean introducing a different answer, which he justifies by returning to the original question and offering a different interpretation. Tim delays dealing with this interpretation until turn 50, and instead asks Steven what he thinks the missing mark could be. Steven gives this answer in turn 48 and Tim repeats it, but rephrasing it as a question. This uncertainty in Tim's repeat de-emphasises the importance of the actual answer. Tim then continues with an explanation of why 79 is the answer. He then continues his turn by talking in general about solving 'mean' type problems, and implying a relationship between multiplying and dividing but he is not specific about what he means by this type of problem or the relevance or when or why you would use multiplication or division (particularly since this problem also involved adding). He then summarises 'what we really did' before returning to Sean's new interpretation of the question. In turn 50 Sean repeats what he said in turn 46. Tim reinforces that this is a new interpretation of the question in turn 51, and builds on Sean's idea that it could be a different average by specifically naming the three averages that the class has been working on.

Extract 6: Introducing different averages

46	Sean:	it could also be seventy one just because the question doesn't specify which aver[age
47	Tim:	[hold on]. we're going back to that, we will go back to that. let's do this one. so, what is that number.
48	Steven:	seventy nine percent
49	Tim:	seventy nine is it? so by by that sort of logic, that there would have to be seventy nine percent. Seventy nine, at least, because seventy nine is the number when that you add up the five numbers and divide by five, that's the one that gives you seventy percent. yeah? is everyone, does, is everyone happy with that. so you know when you do the mean you do it by dividing, quite often when you're solving these (.) mean type problems you end up timesing. wha- what we really did was say, mystery number divided by five is seventy, so what is five times seventy. five times seventy is three hundred and fifty so that's what the total must add up to. yeah? I think we'll wait a few lessons. I think we'll try another one like this and make sure next time everyone can get it. (1.1) right. (0.9) oh what were you going to say
50	Sean:	it could also be seventy one because it doesn't specify which average it is.
51	Tim:	right I've got a feeling that Michelle's parents au- are not perhaps quite au fait with gcse mathematics. they've just said her average test score, and its they haven't actually specified whether they're talking about the mean, or the mode or the median or perhaps some other type of average. okay so we didn't actually say the mean average. it was just an average. so- oh what are you saying it could be
52	Sean:	for the mode it could be seventy one
53	Tim:	she could just get seventy one. (1.1) if she got seventy one percent, she could say to her parents well my average test mark, the mode, is seventy one? (1.6) °where's my surf board.° and what else could it have been actually (0.8) Sophie.
54	Sophie:	it could have been seventy, because then (0.5) er: then you (cra- inaudible) for the median, cause then you get rid of eighty five and forty three,
55	Tim:	well that's sort of what ↑I was thinking y[eah so]
56	Sophie:	[you get ri]d of seventy two and seventy one and then [seventy's in the middle]
57	Tim:	[well if you put the]se a:ll, if you put these numbers in order smallest to biggest, seventy three, forty three, seventy one, seventy two, eighty five. ↑I sort of thought this! what would happen if she's got (0.5) one percent on her (0.6) on her maths mark maths test. what would the averag- the m- median be?
58	Sophie:	it would be seventy one
59	Tim:	it would be the middle one which is seventy one so in a way I don't think it matters (0.6) what she gets in her maths test, because the me:dian will always be (0.4) at least seventy one. if she got a hundred percent over here, the median would move wouldn't it, the median would go up to seventy two. same sort of thing. so ↑in a way she could say, if she really understood the me:dian, you could argue doesn't matter what she got really,

The interaction continues with Sean offering the mode and Sophie offering the median. No detail is given as to how Sean came to his answer of seventy-one, but Sophie

describes the process she used to find the median. Sophie's answer of seventy is only one possible answer for the missing mark that would result in a median of seventy-one but it is Tim in turn 59 who indicates that any other mark would give a median of at least 71. The discussion around the problem ends with Tim's turn, which interprets the median within the context of the problem.

5. Discussion

The analysis above illustrates two ways of working on a problem as a class. Tyler's problem begins his lesson and he uses it to introduce the new topic of probability. Tyler includes at least one 'problem' in all of his lessons and in each case there is a context that the students are able to explore for themselves, either as individuals, or as a whole class, before he brings the class together to introduce the mathematics. Four of these problems are used to introduce a new topic at the beginning of the lesson. Whilst the problem discussed above is unusual in that Tyler does not allow the students to work on the problem independently before working through the solution as a whole class, it still illustrates the nature of the actions of students and teachers in the whole class discussions on solving problems. Tim only includes one task he describes as a problem in his lessons which is the one discussed above. This task comes at the end of a topic on calculating averages. Thus the two problems are serving different purposes, the first introduces the new topic whilst the second is used to apply something the students have learnt.

The discussion around problem solving in Tyler's lesson involved performing calculations, making connections, and interpreting solutions. Yet what the students actually do when working on the problem is perform single step calculations that are identified by the teacher, and on only one occasion identify the feature of the problem that affects what they need to find out. It is the teacher who identifies the appropriate calculations and sequencing of these calculations. It is the teacher who makes the connections between the calculations, and between the calculations and the problem context. It is the teacher who makes the connection between the different solutions and considers the implications for the problem. It is also the teacher who identifies when the problem is solved and summarises and draws conclusions at this point.

The discussion around problem solving in Tim's lesson also involves performing calculations, making connections, and interpreting solutions. In this context the students do more than just perform or report calculations and procedures. It is the students who initially interpret the question, in different ways, and Tim sometimes revoices and sometimes repeats these interpretations. It is the students who have decided which calculations to perform. The students also interpret their solutions in terms of the mathematical concepts they are working with, but it is Tim who makes the connections to the original problem context.

Both teachers state in the introduction of the problem what mathematics will be relevant – probabilities and likelihood for Tyler and mean, mode, and median for Tim. Both teachers use a sequence of questions and prompts that are similar to Wood's (1988) funnelling pattern in that the teachers are directing the topic of the interaction towards their goal which they treat as a solution to the problem.

Returning to the actions that problem solving entails (Guberman & Leikin, 2013; Mason et al., 2010; Schoenfeld, 1992) in both examples students are given the opportunity to understand and make sense of the problem. In Tyler's class this is by playing the game and developing a strategy for winning, and in Tim's class this is by working on the problem independently without teacher support. In both examples, the

teachers make connections to the mathematical ideas they will need, probability in Tyler's case and different averages in Tim's case. Both teachers talk about the problem and represent it with numbers on the whiteboard. Tyler also includes physical materials and a table to remind students of the work they did on calculating the probability of two events. Only Tim makes a connection to similar problems, though this is a generic reference to mean-type problems rather than to any similarities or features of problems where what they have done on the problem may be relevant.

Whilst the students may have engaged with many of these actions individually, only the opportunity to engage with some of these problem solving processes is given. The teachers model many of these actions, but rarely make this modelling explicit and offer few opportunities for the students to engage in problem solving actions or practices themselves within the interaction. Whilst teachers modelling problem solving can support students' development of their problem solving skills, students also need to participate in the actions of problem solving, including the metacognitive skills associated with problem solving (Coles, 2013). Teachers can never guarantee that the intended learning will occur, but they can ensure that the opportunity for students to learn what is intended is available (Marton & Tsui, 2004).

By considering everyday classroom interactions surrounding problem solving, as opposed to lessons involving problem solving overtly, and using examples that typify solving problems in secondary mathematics classrooms, we have demonstrated the limited experiences of problem solving that students have access to. Whilst students were participating actively in the classroom interactions, in particular, we have pointed out this deficit in problem solving opportunities by highlighting the instructional practices that (unsuccessfully) attempted to engage students' thinking, many of which are advocated in the research considered above into teachers' discursive practices. It is worth mentioning that many of these teachers may have specific lessons on problem solving and chose not to share videos of these lessons with us, this in itself treats problem solving as something separate and special. It stands to reason that if problem solving is a fundamental part of learning mathematics, then we would argue that the norms of interaction in all mathematics classrooms should enable students to experience the wide range of skills, actions and problems that many researchers in mathematics education would consider essential to problem solving.

Transcript notation from Sidnell (2010)

Convention	Name	Use
[text]	Brackets	Indicates the start and end points of overlapping speech
(0.5)	Timed silence	Indicates the length, in seconds, of a silence
(.)	Micropause	A hearable pause, usually less than 0.2 seconds
.	Period	Indicates falling pitch or intonation
? or ↑	Question mark or Up arrow	Indicates rising pitch or intonation
,	Comma	Indicates a temporary rise or fall in intonation
-	Hyphen	Indicates an abrupt halt or interruption in utterance
°	Degree symbol	Indicates quiet speech

Underline	Underlined text	Indicates the speaker is emphasizing or stressing the speech
:::	Colon(s)	Indicates prolongation of a sound
(text)	Parentheses	Speech which is unclear in the transcript

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Experiences of problem solving in whole class interactions

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Problem solving is often considered to be an essential part of learning mathematics. Mathematical problem solving involves a range of activities and actions such as making sense of the problem, making connections between the problem and what is known, and using a range of representations. The question then arises as to what experiences students have of these activities in their everyday mathematics lessons. In this paper we examine the whole class interactions around problems and problem solving as they naturally occur in mathematics classrooms. Thus, we are examining students' ordinary experiences of problem solving in their everyday mathematics lessons. This contrasts with existing research that focuses on students' activity around cognitively demanding tasks, or in lessons specifically designed to focus on problem solving. Using a Conversation Analytic approach we examine the whole class interactions around the introduction of tasks described by the teacher themselves as problems, and the subsequent whole class interactions that followed where the class worked on the problem together. The data comes from two lessons from a corpus of 52 lessons taught by 17 teachers in 8 schools. The examples were chosen as they serve to illustrate the typical and ordinary treatment of problem solving in ordinary mathematics classroom interactions. Our analysis shows how students participate in a very narrow range of problem solving actions and that the actions that they do participate in are tightly controlled by the teacher's practices. Whilst the teachers model many of the problem solving processes, they do not draw attention to these processes or use them to scaffold students' future use of them. This is despite the teachers using a range of discursive practices, such as revoicing, that previous research has advocated as a tool for engaging students in mathematical practices such as problem solving. The analysis reveals the intricate way in which teachers' discursive practices influence and shape students' experiences of learning mathematics. This raises implications for the development of mathematics teaching practices that consider the sequential role of discursive practices such as revoicing, rather than moves in isolation. The analysis also raises implications for what students perceive and interpret problem solving to be in mathematics.