# Mathematical modelling competence. Selected current research developments

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**Mathematical modelling competence. Selected current research developments**

***Abstract***

*Current research areas in the field of mathematical modelling are identified on the basis of specific research and development projects. Modelling cycles are an important theoretical basis for this. The measurement of students modelling competence as well as that of competence for teaching mathematical modelling with the help of written tests are key components. The investigation of different mathematical modelling tools, such as the use of technology in larger control group studies, and the evaluation of seminars in teacher education, are current lines of research in the field of modelling in mathematics education. Technology use in mathematical modelling is given special consideration. Overall, selected studies from Germany are used as examples to provide insight into the current research landscape.*

**Keywords.** Mathematical modelling; modelling cycle; competence; technology; tests.

**Competencia en modelización matemática. Selección de desarrollos actuales de investigación**

***Resumen***

*Las líneas de investigación actuales en el ámbito de modelización matemática se identifican en base a los proyectos específicos de investigación. Los ciclos de modelización son una aproximación teórica importante para ello. La medición de la competencia de modelización de los estudiantes, así como la de las competencias para la enseñanza de la modelización matemática con la ayuda de pruebas escritas, son componentes empíricas esenciales. La investigación desarrollada sobre diferentes herramientas de modelización, como el uso de la tecnología en investigaciones con grupos control más grandes, y la evaluación de distintos cursos en la formación del profesorado, son líneas de investigación actuales en el ámbito de la modelización. Se presta especial atención a la utilización de la tecnología en la elaboración de modelos matemáticos. En este trabajo, se escogen algunos ejemplos de investigaciones desarrolladas en Alemania para dar idea del panorama actual de investigación.*

**Palabras clave.** Modelización matemática; ciclo de modelización; competencia; tecnología; pruebas.

## 1. Introduction

Research on the teaching and learning of mathematical modelling contains a strong emphasis on developing local theories (Geiger & Frejd, 2015). One of such current theories deals with modelling competence. Many aspects of these theoretical considerations have an influence on empirical studies conducted in various fields. In particular, the learning and teaching of mathematical modelling are important current lines of development (Stillman, 2019).

The aim of this paper is to consider the spectrum of research in Germany on modelling competence and to demonstrate research instruments and findings in this field. I focus on some German research projects on the modelling competence of both learners and teachers. Nowadays, an application orientation is a natural component of mathematics classes and educational standards in Germany. Modelling has been included as a competence in the educational standards (KMK, 2012) and the curricula of the various federal states. There are many research projects on mathematical modelling with a significant increase in the past ten years. In addition to this increase, there have also been notable methodological developments. This could be one reason for the changes of the type of research projects carried out on mathematical modelling in the past few decades. Research projects on modelling competence evaluation now more commonly use experimental control group designs and sophisticated statistical methods to analyse various research questions. In particular, the use of technology in mathematical modelling is increasingly considered in research projects. Some research results on mathematical modelling are presented below by way of examples.

## 2. Theoretical background: Modelling cycles and modelling competence

The entire modelling process is often presented in an idealised version as a modelling cycle. Idealised means that this representation itself is also a model. The literature therefore contains various cycle representations of modelling. Just such a model was created by Blum and Leiss (2007) from a cognitive perspective (see Figure 1). For this purpose, a model previously created by Blum (1985) and developed further by various researchers was extended by the situation model. The creation of the mathematical model is addressed in detail and the process of the individual creating the model is set out in greater detail in this modelling cycle. The situation model describes the mental representation of the specific situation by the individual.

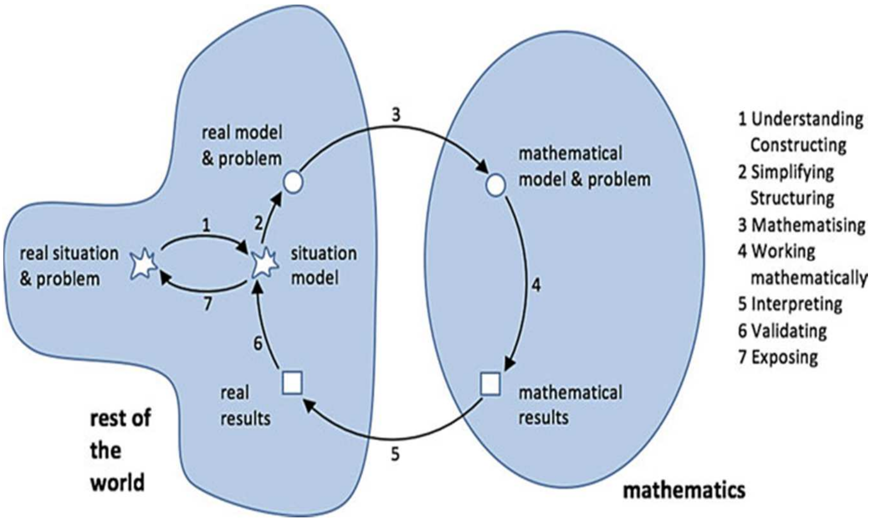


Figure 1. *Modelling cycle according to Blum and Leiss (2007, p. 225)*

This modelling cycle (Figure 1) describes the various sub-processes of modelling more accurately and in more detail than many other modelling cycles. Therefore, we use this cycle for our further considerations. The ability to perform such a sub-process can be seen as a specific modelling competence (Kaiser, 2007; Maass, 2004). These competencies could be characterised as presented in Table 1. *Competence* is here used in a broader sense whereas *competency* refers to the different constituents of competence (Blömeke et al., 2015). By means of detailed descriptions the nature of competencies becomes obvious, so that an extensive list of modelling competencies can be obtained. Working mathematically has been included in the list of competencies for the sake of completeness, because working mathematically is also a sub-process in the modelling cycle. However, it should be remembered that working mathematically is not as typical for modelling processes as mathematizing, for example. While mathematising plays no role in few other mathematical competences such as problem solving or proving, it is different in working mathematically. By using different modelling cycles, other competencies emphasising other aspects of modelling could occur (Greefrath & Vorhölter 2016).

Table 1. *Competencies in modelling (Greefrath et al., 2013, p. 19; Greefrath & Vorhölter, 2016)*

| Competency | Description |
|---|---|
| Constructing | Students construct their own mental model from a given problem and thus formulate an understanding of their problem. |
| Simplifying | Students identify relevant and irrelevant information from a real problem. |
| Mathematizing | Students translate specific, simplified real situations into mathematical models (e.g., terms, equations, figures, diagrams, and functions). |
| Working mathematically | Students work with mathematical methods in the mathematical model and obtain mathematical solutions. |
| Interpreting | Students relate results obtained from manipulation within the model to the real situation and thus obtain real results. |
| Validating | Students judge the real results in terms of plausibility. |
| Exposing | Students relate the results obtained in the situational model to the real situation, and thus obtain an answer to the problem. |

## 2. Measurement of students' modelling competence

The aim of several research projects to assess and control the modelling competence of learners (e.g., Schukajlow et al., 2015). The assessment of such competence always depends on the underlying concept of competence. Modelling competence not only involves the ability to model, but also the willingness to address problems with mathematical aspects from reality, using mathematical modelling (Kaiser, 2007, p. 110). It is therefore difficult to develop a written test for measuring modelling competence that takes this into account. It is necessary not only to check knowledge, but to measure modelling competence in specific situations. In order to do this, a written test was developed that confronts students with a selection of situations that can be processed using mathematical methods (Hankeln et al., 2019).

In constructing a test, it is necessary to decide whether to use holistic or atomistic modelling tasks (Blomhøj & Jensen, 2003). Holistic tasks require a full modelling cycle to be carried out, while atomistic tasks are pre-constructed and focus on just one or two steps in the modelling process. The use of holistic tasks is appropriate when measuring general modelling competence, which has already been done in various studies (Kreckler, 2017; Rellensmann et al., 2017; Schukajlow et al., 2015). In atomistic tasks students need only to process problems that require a limited range of modelling competence. These tasks cannot be used to obtain information about whether a person would generally be able to carry out a full modelling process. However, atomistic tasks can be used to measure different modelling competencies separately from one another, which is not possible with holistic tasks. There are already tests that use atomistic modelling tasks, but these only summarise various competencies (Brand, 2014; Zöttl et al., 2011). One of the first tests with atomistic modelling tasks was developed by Haines, Crouch and Davis (2001) and served as a reference point for further developments. It originally consisted of 12 items, each with five possible answers.

Hankeln et al. (2019) constructed a test that records the competencies of simplifying, mathematizing, interpreting and validating separately. Holistic tasks are not used, due to the large number of test items that would be required. One example of an item for the competency of simplifying is proposed in the lighthouse task of Figure 2. The students' task is to select all information that is relevant to calculate the distance to the horizon.

This multiple-choice item is thought to measure the competency of identifying relevant quantities and key variables. This is part of the definition of the competency of simplifying. Corresponding items for the other competencies were also developed. There are pre-tests and post-tests, each with two groups in a multi-matrix design. Each test booklet consists of 16 items and takes roughly 45 minutes to complete. An evaluation of the test instrument with 3300 student responses to the test was able to show that the data collected can best be described using a four-dimensional between-item model, in which the various competencies are recorded as separate dimensions of a latent construct. This result shows with what level of certainty the modelling competencies at play can be measured empirically. It was also possible to conclude that the competencies of simplifying, mathematizing, interpreting and validating can be understood as different components of a global modelling competence (Hankeln et al., 2019).

**During their summer vacation, Marcus and Irina are standing on top of a lighthouse and enjoying the view. "How far is it to the horizon?" Irina asks.**

***Mark all of the following information that you consider to be important to calculating the distance to the horizon.***

https://upload.wikimedia.org/wikipedia/commons/b/bf/Louisbourg_Lighthouse.jpg

| | | | |
|---|---|---|---|
| ☐ | Between the lighthouse and the ocean, there are 25 m of sandy beach. | ☐ | The two are standing on the Atlantic coast in France. |
| ☐ | There are no clouds in the sky. | ☐ | The radius of the earth measures 6370 km. |
| ☐ | The lighthouse is 83 m high. | ☐ | The lighthouse's light shines as far as 10 km. |

Figure 2. *The lighthouse task (translated): multiple-choice item that measures the competency in simplifying a problem* (Hankeln et al., 2019, p. 148)

## 3. Promotion of students' modelling competence

Some research focuses on how to promote modelling competence in school using various different tools. One example of a project that took into account the investigation of the promotion of modelling competence is the LIMo project at the University of Münster (2015-2018). The aim of the project was to investigate whether modelling competence can be promoted using *digital* tools such as dynamic geometry software and *strategic* tools such as a solution plan. In order to do this, an interventional study was carried out in spring 2016 in a quasi-experimental pre/post/follow-up design in 44 grade nine classes in German grammar schools, and the development of student competence was measured using a previously developed modelling test with items testing the competencies. The intervention consisted of a series of four class sessions (each of 45 minutes) on modelling tasks. During the class, students had to calculate, for example, the lawn area of a castle garden. A sketch of the castle garden was available for this purpose (see Figure 3). The students initially had to discuss which green areas belonged to the castle garden and what simplifications they could make to calculate the area.

The 44 classes were broken down into three groups of approximately the same size. All of the groups worked on the same modelling tasks, with one being completed in each session. One group also used dynamic geometry software (GeoGebra), the second group used a five-step strategic solution plan with cognitive learning strategies in each step in the modelling process that was available on posters and worksheets for the entire investigation, and the third group used neither of these tools.

Strategic solution plans are often based on steps in the modelling process or competencies that play a key role in modelling. As part of the DISUM project (Blum & Leiss, 2007), a solution plan was developed for students, based on a simplified modelling cycle. This solution plan comprises four steps: understanding the task, creating the model, using mathematics, and explaining the result. Each step is explained to students using a question and a number of explanatory points. In a study by Schukajlow et al. (2015) as part of the DISUM project, significant differences in student performance were demonstrated when modelling with this solution plan, with reference to the content area of "Pythagoras' theorem". Teaching using the solution plan proved to be a more effective form of teaching and learning. In addition, students in the solution-plan group also perceived a greater use of the solution plan. A five-step solution plan was used in the LIMo project at the University of Münster. This solution plan comprises the following steps: 1) Understanding and simplifying, 2) Mathematizing, 3) Working mathematically, 4) Interpreting and 5) Controlling. These five steps were selected to highlight in particular the step of validating the result and determining the path to a solution. Only a short-term improvement entailing a small effect in performance for the competencies of interpreting and validating could be identified for the strategic solution plan for the entire sample, consisting of both test groups investigated with and without a solution plan. The investigation of group membership as a factor for the development of competence showed that the solution plan has only a minor effect on the development of the interpreting competency, while no interaction effect between the test group and the time of measurement could be identified for the other competencies. In terms of long-term competence development, with a further measurement point defined three months after the class sessions, there was a long-term, stable increase in the interpreting competency in the solution plan group (Beckschulte, 2019).

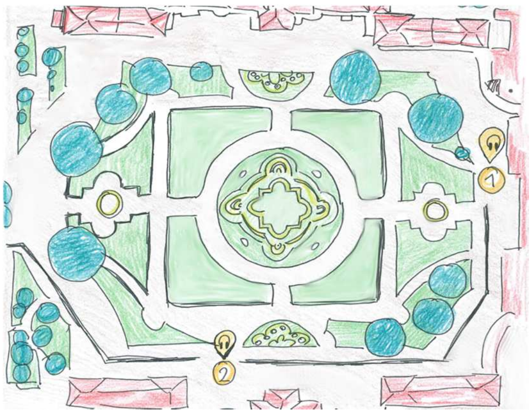

Figure 3. *Sketch of the castle grounds* (Hankeln, 2018, p. 152)

In terms of the use of the dynamic geometry software, we separate the modelling competence from the competence to use the software. It was assumed that it was not the modelling competencies of the students at the first measurement point that impacted on the effectiveness of the intervention, with or without dynamic geometry software, but rather their competence in using the software. This assumption was confirmed in that no significant interaction effects could be identified between the competencies at the first measurement point and the test group. The class unit on dynamic geometry software had an equal effect on the competency development of both students who were initially stronger and those who were initially weaker. The analysis of the data, however, showed that the test group factor did not have a significant impact in any of the competencies. Contrary to expectations, the competencies did not differ accordingly (Hankeln, 2018).

## 4. Students' Modelling competence and technology

Looking at modelling processes with technology, the list of competencies of modelling can be extended. The various uses of technology are effective in different parts of the modelling cycle, working on application-oriented tasks. Validating, for example, is an activity that can be supported effectively by technology. Control processes generally belong to the final stage of the cycle (e.g., Figure 1, step 6). Calculations with technology are carried out using the mathematical model created, which, for example, is a function in calculus (e.g., Figure 1, step 4). A more precise analysis shows that technology can be used sensibly and meaningfully when modelling in all phases of the modelling cycle.

A look at the step of working mathematically (Figure 1, step 4) in greater detail, reveals that the processing of modelling problems using technology requires two translation processes. The modelling task first needs to be understood, simplified and translated into the language of mathematics. Technology can, however, only be used when the mathematical expressions have been translated into the language of the computer, and a computer model has been developed. The results from using technology then have to be transformed back into the language of mathematics. Ultimately, the original problem can be solved if the mathematical results relate to the real situation. These translation processes can be set out in an expanded modelling cycle (see Figure 4) which, in addition to the real world ("rest of the world") and mathematics, also takes into account the technology (see Savelsbergh et al., 2008). Accordingly, the list of competencies (Table 1) is extended by finding the computer model ("technologizing") and interpreting the computer results in the mathematical world ("transferring").

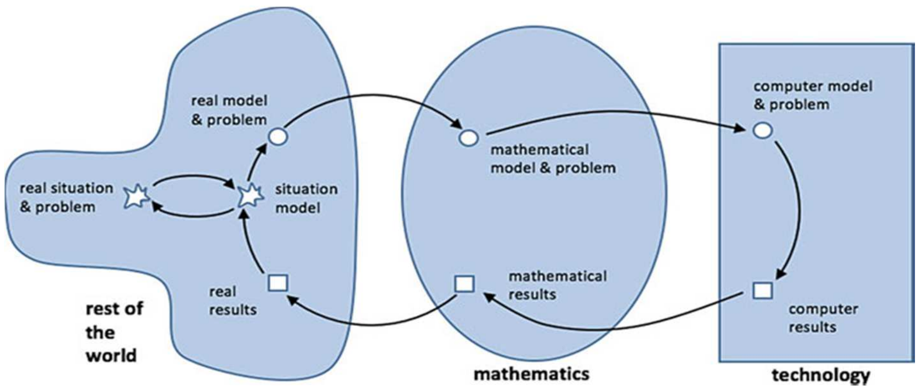


Figure 4. *Possible use of technology in modelling cycle* (Greefrath, 2011, p. 303)

In Hankeln (2018), no clear evidence was found that the use of digital media can be useful for student modelling processes. The link between software-related self-efficacy, the mathematization competency and beliefs regarding the dynamic geometry software were analysed within the group with dynamic geometry software. There is a significant correlation between software-related self-efficacy and beliefs about the software. Students who felt more confident about their competence with the tool rated the software more positively and vice versa. It was also possible to show that the software-related self-efficacy was a significant predictor of mathematization competency in the post-test, even when we controlled for the pre-test. Students with higher software-related self-efficacy improved their mathematization competency more than those with lower self-efficacy, albeit with a small effect size (Greefrath, Hertleif, & Siller, 2018).

In a case study with four pairs of students in grade 10 at a grammar school in Germany, Greefrath and Siller (2017) observed students working on a reality-based task with GeoGebra. The researchers were interested in which processes in the modelling cycle technology were used and which activities were performed with technology during modelling. The use of technology took place mainly in mathematizing and working mathematically. In addition, there was also technology usage between the situation model and the mathematical model. The observations show that technology is used at different processes in the modelling cycle (Greefrath & Siller, 2017)

A number of other studies that provide more detail on modelling competence with technology. Doerr and Zangor (2000, p. 151) examined learners in two classes of upper secondary education with graphic calculators over 6 lessons. It was shown that the technology was used as a “transformational tool”, for calculation, data acquisition, visualization and control. Accordingly, Arzarello et al. (2012) showed that technology such as GeoGebra is used by students for testing assumptions and for validation. Geiger et al. (2003) and Brown (2015) also showed that students need significant support to use technology beyond mathematical work. It is therefore not self-evident that students use technology in a variety of ways. In principle, modelling competence in connection with technology can obviously be documented and described in the modelling cycle.

## 5. Measurement of competence in teaching mathematical modelling

In order to promote the development of modelling competence of students, it is useful to look at professional teacher competence in mathematical modelling. Teachers' professional competence can be described using various models based on Shulman (1986), in which the core areas of teacher competence are described. Based on the model used in the COACTIV study (Baumert & Kunter, 2013) and the theoretically derived dimensions of competence from Borromeo Ferri and Blum (2009), a model was developed specifically for teaching mathematical modelling.

Certain aspects and areas of competence were selected from the COACTIV model (Baumert & Kunter, 2013) with a focus on the teaching of mathematical modelling. In the field of professional knowledge, pedagogical content knowledge is characterised by specific content with regard to teaching mathematical modelling. Beliefs and self-efficacy can also be specified in this context. Pedagogical content knowledge was split into four areas of competence, taking into account the dimensions of competence of Borromeo Ferri and Blum (2009). These include knowledge of interventions, modelling processes, modelling tasks and modelling goals. Diagnostic competence concerning knowledge of modelling processes, for example, consists of the ability to identify modelling phases and the ability to recognise difficulties in the modelling process. A quantitative test instrument on teaching mathematical modelling was developed on the basis of the structural model. The test consists of two parts. In the first, modelling-specific pedagogical content knowledge is recorded in a performance test. In order to do this, 70 dichotomous test items were operationalised in a multiple choice and combined single-choice format. The items in the fields of knowledge about modelling processes and knowledge about interventions relate to modelling tasks that are supplemented with text vignettes on specific solution processes of students (example item, see Figure 5).

In the second part of the questionnaire, beliefs and self-efficacy with regard to mathematical modelling are recorded on five scales. Abbreviated scales on constructivist and transmissive beliefs about teaching and learning in mathematics, based on Staub and Stern (2002), were used. The scale on the application aspect from Grigutsch, Ratz and Turner (1998) was adapted to the content of mathematical modelling. A scale representing the use of mathematical modelling in the classroom was developed. A newly developed instrument which focuses on self-efficacy in perceptions and rating of performance heterogeneity was used to determine expectations of self-efficacy. The full test has been published in Klock and Wess (2018).

When piloting the test instrument, data from 156 student teachers (66.9% female) at various universities in Germany were collected. At time which the data were collected, the students were either at the end of their Bachelor's degree (12.7%) or doing a Master's degree (87.3%). The results of the pilot study show that the structural model for teaching mathematical modelling in this form was able to be confirmed empirically. Only the scale of transmissive beliefs about the learning and teaching of mathematics showed no significant charge or explained variation (Klock et al., 2019).

**Example item text vignette: container (grade 8)**

Containers are used to store construction materials or collect construction waste on many construction sites. These containers have a specific shape to facilitate loading and unloading. How much sand is in the container shown?

STUDENT 1: It contains exactly 7,160,000 cubic metres of sand. Can that be right?

STUDENT 2: It could well be right, you calculated it with your calculator.

STUDENT 1: Well yes. Then it's right.

STUDENT 3: It's definitely right. I can imagine it.

Which phase of the solution process are the students mainly in? Please mark accordingly.

Mathematising □
Working mathematically □
Interpreting □
Validating □

Diagnose the students' problem with working through the task in this situation. Please mark accordingly.

The students...

...have problems making assumptions. □
...are not checking their solution sufficiently for plausibility. □
...are drawing an incorrect conclusion from their mathematical result. □
...are using an unsuitable mathematical model. □

Figure 5. *Example item text vignette* (Klock & Wess, 2018, p. 22)

## 6. Promotion of student teachers' competence in teaching mathematical modelling

For promoting the competence in teaching mathematical modelling of student teachers, practical elements of mathematical modelling are most suitable. The use of teaching and learning laboratories enables the inclusion of practical elements in teacher training at an earlier stage. An important goal of teaching and learning laboratories is the professionalization of future teachers through reflection on the teaching and learning process (Putnam & Borko, 2000). The teaching and learning laboratory MiRA+, specialising in modelling, was developed at the University of Münster. It is integrated into the training for grammar school teachers and consists of a seminar with 12 seminar sessions and additional blended learning formats in the design of modelling tasks. The seminar consists of a theory-based preparatory phase, a practical phase and a reflection phase. The key element in terms of content of all phases consists of modelling processes and potential-oriented handling of heterogeneity.

The preparatory phase of the seminar looks at selected backgrounds of mathematical modelling (modelling cycle, modelling competencies) and the students' own work on a modelling task. An example of a modelling task used in the seminar is illustrated in Figure 6. Individual support is discussed in connection with a productive way of handling heterogeneity. Based on this, criteria for suitable modelling tasks are then created, and tasks of this type developed by the student teachers as part of a blended learning format with various feedback cycles for use in the practical phase. Criteria and indicators on specific individual processes of modelling are then created to monitor and diagnose the students' learning processes in the teaching and learning laboratory sessions. The development of modelling tasks and the creation of a suitable catalogue of criteria which deals intensively with the diagnostic individual processes forms the basis for promoting competence in teaching mathematical modelling. In the practical phase, a team of three student teachers (Master of Education) supports a small group of grade nine students with the processing of the modelling tasks they have created during the 90-minute project sessions. The teams monitor the competencies of mathematical modelling in a targeted manner and record the results in the previously created monitoring sheet. The grade nine students work on content that enhances the curriculum by motivating project contexts. This interlacing of theory and practice in the context of diagnostic actions and tasks represents the practical promotion of modelling-specific diagnostic and task-based competence.

Figure 6. *Hot air balloon task: "How many litres of air are in this hot air balloon?"*

During the reflection phase, the project sessions are first discussed in the form of written reflection discussions, so that student teachers can benefit from the experiences of other seminar participants. Cross-task, theory-based group reflections on the respective areas of focus of the monitoring are carried out, taking into account in particular the heterogeneity aspects of the learning groups monitored. The student teachers supplement their diagnostic assessments with feedback from their colleagues. The knowledge obtained is then used to professionalise the participants' own teaching activities and to evaluate the modelling tasks they created. The student teachers also reflect on, and where necessary, adapt the modelling tasks in light of the criteria for good modelling tasks formulated in the preparatory phase. The experience and knowledge gained are summarised in a reflection report.

As part of the study, an investigation was carried out to determine the extent to which aspects of the modelling-specific diagnostic and task-based competence can be promoted among future teachers in the mathematical teaching and learning laboratory MiRA+. Data were collected from 96 student teachers using a pencil and paper test in the pre-post design (Klock et al., 2019). In addition to the experimental group at the University of Münster (N = 35) and the comparison group at the University of Koblenz-

Landau (N = 43) where they used predefined modelling tasks, a baseline group in Münster (N = 18) was also recorded to control the test repetition effect. It was evident that the experimental group improved significantly with a major effect on the three aspects of development, analysis and multiple solutions of task-based competence, while the comparison group from Koblenz only showed significant improvements with a moderate effect on the aspect of analysing of modelling tasks, and for the baseline group, no significant changes. In terms of modelling-specific diagnostic competence, both the experimental group and the comparison group showed significant improvements over time, with a major and moderate effect respectively for the aspects of identifying the modelling phase and difficulties in the modelling process, while the baseline group once again showed no significant changes. These observed increases can be attributed primarily to the different priorities at the participating locations. The investigation of modeling-specific diagnostic and task competence, which have a strong influence on the acquisition of competence in teaching mathematical modelling by students, thus provides a clear indication that professional competence in teaching mathematical modelling has been promoted successfully both in the context of the MiRA+ teaching and learning laboratory, and in the comparison group (Wess & Greefrath, 2020).

## 7. Summary and outlook

Using German studies as an example, the various projects selected show that there are different lines of research on modelling competence. On the one hand, instruments are being developed with which the modelling competences of students and teachers can be measured quantitatively. There are also very interesting and useful qualitative studies that deal with the modelling competence of students. In this context, the observation of technology use in modelling is only one example. On the other hand, there are various projects that aim to promote modelling competence with the help of various tools –in the examples, I addressed solution plans and digital tools. Technology can influence modelling processes in a unique way. This is shown additionally by several qualitative studies. Also, promising approaches to promoting professional competence for teaching mathematical modelling to student teachers and teaching staff could be reported.

The empirical results presented show some areas of research focus on modelling and application in the past few years in Germany. New test instruments provide opportunities for research on and the development of teaching and learning. The impact of technology on school practice and research projects on mathematical modelling is generally regarded as an important task. Effective promotion of modelling competence among students and the professionalization of future teachers are currently the core elements of research. At the same time, tools and strategies are being researched and developed to help students to model problems independently and train teachers to teach mathematical modelling (Greefrath & Vorhölter, 2016; Barquero et al., 2017). The increase in competence of student teachers in the teaching and learning laboratory for modelling, especially through modelling tasks with digital media created there, is promising. In the future, technology could offer new impetuses for mathematical modelling, both for classes in school and for research methods at universities.

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# Mathematical modelling competence. Selected current research developments

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Current research areas in the field of mathematical modelling are identified on the basis of specific research and development projects. Modelling cycles with and without possible use of technology are an important theoretical basis for this. They describe the various sub-processes of modelling in detail. The ability to perform such a sub-process can be seen as a specific modelling competence. The measurement of students modelling competence with the help of written tests are key components of German research projects. Atomistic tasks were used to measure different modelling competencies, namely simplifying, mathematizing, interpreting and validating, separately from one another. The aim of the LIMo project in 44 grade nine classes in German grammar schools was to investigate whether modelling competence can be promoted using digital tools such as dynamic geometry software and strategic tools such as a solution plan. With respect to long-term competence development there was a long-term, stable increase in the interpreting competency only in the group with a solution plan. But it was possible to show the software-related self-efficacy as a significant predictor of mathematization competency. Further observations in a case study show that technology is used at different processes in the modelling cycle. Another quantitative test instrument on teaching mathematical modelling was developed on the basis of a structural model. The test consists of two parts: in the first, modelling-specific pedagogical content knowledge is recorded in a performance test; in the second, beliefs and self-efficacy regarding mathematical modelling are recorded on five scales. The results of the pilot study show that the structural model for teaching mathematical modelling was able to be confirmed empirically. For promoting the competence in teaching mathematical modelling of student teachers, practical elements of mathematical modelling are most suitable. The use of teaching and learning laboratories enables the inclusion of practical elements in teacher training at an earlier stage. The teaching and learning laboratory MiRA+, specialising in modelling, was developed at the University of Münster. An investigation determined the extent to which aspects of the modelling-specific diagnostic and task-based competence can be promoted among future teachers in the mathematical teaching and learning laboratory MiRA+. It was evident that the experimental group improved significantly with a major effect on the three aspects of development, analysis and multiple solutions of task-based competence. With respect to modelling-specific diagnostic competence, the experimental group showed significant improvements over time, with a major effect for the aspects of identifying the modelling phase and difficulties in the modelling process. The investigation thus clearly indicates that professional competence in teaching mathematical modelling has been promoted successfully in the context of the MiRA+ laboratory. Overall, selected studies from Germany are used as examples to provide insight into the current research landscape.