Introduction to 'Research on the teaching and learning of mathematical modelling: Approaches for its design, implementation and analysis'

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1. Introduction

Mathematical modelling has become, in recent decades, an important research topic for the teaching and learning of mathematics. If we focus on both research and its implementation in classrooms, there has been clear progress (Blum, 2015; Burkhardt, 2018) with the transposition of conceptualizations and recommendations on mathematical modelling and its teaching, to school realities at different educational levels. Despite these developments, there are still many open questions that affect both realities, research and practice, such as, for instance, difficulties of long-term diffusion of modelling in the classroom, challenges for its evaluation, difficulties of its classroom management and needs for teacher education (Carreira et al., 2019).

In research, multiple theoretical frameworks have appeared that address the problems of teaching and learning of mathematical modelling. A good example are the two special issues in *ZDM* (2006 (*2-3*), 2018 (*1-2*)) on this topic. A frequently asked question is about clarifying and connecting theoretical approaches to describe, design, and analyse modelling activities (Kaiser & Sriraman, 2006). Faced with such diversity, it seems necessary to discuss how mathematical modelling is conceptualized within a specific research framework; and how research problems and methodologies about the design and analysis of school practices are consequently formulated.

The idea behind this monograph comes from the research seminar 'An international perspective on the teaching and learning of mathematical modelling', realised at the XXIII SEIEM Symposium. In that meeting, the current state of research across countries (Germany, Spain and Portugal) was discussed, and an overview of the evolution of recent European research was given. As an extension of the seminar, this monograph aims to present an international perspective of research on mathematical modelling teaching and learning. It reviews advances from diverse theoretical and methodological approaches and showcases proposals for design and implementation of teaching practices and teacher training courses for teaching mathematical modelling.

The issue is composed of seven articles, which can be grouped into three thematic blocks. Although they are not independent from each other, this structure is meant to make their reading more accessible. The first block presents an overview of the theoretical and methodological principles for design and analysis of modelling activities from different theoretical frameworks: the modelling cycle and competencies approach; the models and modelling perspective; the anthropological theory of the didactic; and, the cultural-historical activity theory. The second block deals with the interaction that mathematical modelling can establish with other processes, such as experimentation, simulation, or problem solving. The third block concludes this issue with a reflection on the consequences of working within a certain theoretical framework, and hence inheriting a concrete way of interpreting mathematical modelling, with its design principles of modelling activities and its analysis tools.

2. Organization and presentation of articles

The first block begins with the work of the researchers Jonas Bergman Ärlebäck (University of Linköping) and Helen M. Doerr (Syracuse University), situated within the theoretical framework of models and modelling perspective. The central goal of their study is to develop designs of *model development sequences* to be implemented in classrooms, and to use these sequences as tools for teachers to guide student activity in model building. They particularly show the design and implementation of a model development sequence, about how light intensity changes with respect to the distance from a light source. Based on this experience, it is analysed the role that the design of the sequence of model development played for the teacher, as well as the strategies for classroom management. Tools are proposed to guide students in the development of models, from being descriptive towards being explanatory.

The work of Ignasi Florensa (Escola Universitària Salesiana de Sarrià–UAB), Javier García (Universidad de Jaén) and Gemma Sala (Universitat de Barcelona) grounds on the anthropological theory of the didactic. They show how mathematical modelling is conceived from this theoretical approach. They describe *study and research paths* (SRP) as didactic-mathematical devices for the teaching of modelling. They analyse three SRPs designed and implemented at three educational levels – Primary, Secondary and University– for an "ecological" analysis of mathematical modelling. This leads them to analyse conditions that favour and restrictions that limit the development of the mathematical modelling activity in the experimentation contexts. They pay special attention to persistent restrictions across school institutions and, in contrast, to conditions that have favoured the development of modelling.

The article of Gilbert Greebrath (University of Muenster) takes the theoretical perspective of the *modelling cycle and competencies*, widely used in the research on modelling and applications. His work describes the current lines of research from this theoretical lens by means of research mainly carried out by the German community. We can observe how, based on the characterization of the modelling cycle and the modelling competence, efforts have focused on two lines of research. The first line focuses on how to promote modelling competence in classroom practice with students, and through teacher training; the second line considers the design of tools and analysis techniques to quantify the degree of development of the modelling competence by the students, or the professional competence for the teaching of modelling.

Pauline Vos (University of Agder) and Peter Frejd (University of Linköping) present their research about how a group of high school students are able to describe and use Sankey diagrams, as objects in themselves and/or as tools for modelling and studying extra-mathematical questions. Their work presents the analysis of this group work across several modelling projects that link school mathematics with the study of demographic or economic phenomena in their region. Their approach to modelling, based on the cultural-historical activity theory (CHAT), defines the *object-tool duality*, which seeks to combine the vision of mathematics as a set of objects and as a set of tools. They conclude about the necessity to prioritize mathematics teaching that puts the tool before the object, to increase the relevance that students give to mathematics.

After this first block on theoretical frameworks, the second block analyses the contact of mathematical modelling with other processes. The first work, by Susana Carreira (University of Algarve & UIDEF), Ana Margarida Baioa (Group of Schools D. Manuel I) and Lourdes Maria Werle de Almeida (State University of Londrina), examines how experimentation and simulation influence meaning production during a

modelling process. To do so, they propose tools that allow a joint look at the modelling process together with the prototyping process, where experimentation and simulation appear as relevant elements in student meaning production. They present the analysis of two experiments realised in Portugal and Brazil, at the Secondary and University levels, of a project regarding the biometric recognition of the hand. The multiplicity of experimental contexts allows them to detect similarities and differences between the ways of approaching the project, the models developed and the influence that experimentation and simulation have had in giving meaning.

It follows the work by Irene Ferrando and Carlos Segura (Universitat de València), who discuss how to promote flexibility and adaptability in problem solving, through a sequence of mathematical modelling tasks. Their research links two connected realities, those of problem solving and of mathematical modelling. They analyse resolution plans and strategies that a group of pre-service Primary school teachers develop to solve modelling tasks. The type of modelling tasks chosen is a sequence of four Fermi problems, for which they make available a categorization of the plans and strategies for solving this sequence of tasks. The analysis of responses from the teachers in training, which combines qualitative and quantitative methods, allows the authors to show how the specific sequence of proposed modelling tasks promotes inter-task and intra-task flexibility, as well as the adaptability of resolution plans.

Finally, the third block includes one article only. Berta Barquero (Universitat de Barcelona) and Britta Eyrich Jessen (University of Copenhagen) discuss how the adoption of a particular theoretical framework affects task design in the research on modelling and applications. It is highlighted the existence of different reference epistemological models, that are constructed from research, to deal with problems related to teaching and learning mathematical modelling. From the discussion of three case studies, corresponding to three theoretical approaches, they analyse the impact of these reference epistemological models on decision-making regarding the design and implementation of modelling tasks. The cases illustrate the dialectic between the theoretical hypotheses within a specific framework and the design principles of modelling activities.

In summary, this monograph includes a collection of works that cover various geographical and theoretical perspectives, as well as combine theoretical and methodological contributions with the design and analysis of implementations at different levels and school contexts. It is our hope that the monograph will provide a vision of current advances in research on mathematical modelling, and that it will stimulate continuous advancing through fruitful dialogue between research, classroom practice and teacher education, to strengthen modelling teaching and learning

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