Design and research for developing local instruction theories

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Diseño e investigación para el desarrollo de teorías de instrucción local

Resumen

La innovación en educación matemática necesita de la implicación de profesores, autores de libros de texto, legisladores e investigadores. Este artículo esboza el papel y la importancia del diseño instruccional orientado hacia nuevas teorías de instrucción local en educación matemática. Muestro esta aproximación con un estudio donde se investigó cómo apoyar a los estudiantes en el desarrollo de los principios básicos de las matemáticas del cambio. El estudio combina diseño e investigación en tres fases sucesivas. En la primera fase se diseña una trayectoria hipotética de aprendizaje y actividades de enseñanza, en la fase del experimento de enseñanza se implementa la trayectoria, y en la fase de análisis retrospectivo se reflexiona sobre las hipótesis adoptadas. Así se estructura un proceso cíclico de (re)diseño y desarrollo de enseñanza innovadora. Se espera que la teoría de instrucción local resultante cree oportunidades para que profesores, autores de libros de texto e investigadores puedan adaptar los resultados a su investigación o práctica de aula, teniendo en cuenta el contexto en el que trabajan.

Palabras clave. Diseño de tareas; teorías de instrucción local; educación matemática; investigación de diseño.

Design and research for developing local instruction theories

Abstract

Innovation in mathematics education needs the involvement of teachers, textbook authors, policy makers and researchers. This paper sketches the role and importance of instructional design aiming at new local instruction theories in mathematics education. The approach is shown with a study that investigated how students can be supported in the development of the basic principles of the mathematics of change. The study combines design and research in three successive phases. In the first phase a hypothetical learning trajectory and instructional activities are designed, in the teaching experiment phase the trajectory is acted out, and in the phase of the retrospective analysis the articulated hypotheses are reflected upon. In this way, a cyclic process of (re)design and development of innovative teaching is structured. The resulting local instruction theory is expected to create opportunities for teachers, textbook authors and researchers to consider contextual factors and to adapt results for their research or teaching.

Keywords. Task design; local instruction theories; mathematics education; design-based research.

Desenho e pesquisa para o desenvolvimento de teorias de instrução local

Resumo

A innovação em educação matemática carece do envolvimento de professores, autores de manuais, legisladores e investigadores. Este artigo esboça o papel e a importância do esquema educacional orientado para as novas teorias de educação local em educação matemática. Apresento esta abordagem mediante um estudo onde se investigou a forma de apoiar os alunos no desenvolvimento de princípios básicos da matemática para a mudança. O estudo combina esquematização e investigação em três fases

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sucessivas. Na primeira fase esquematiza-se um percurso hipotético de aprendizagem e atividades de ensino, na fase de experiencia de ensino implementa-se o percurso e, na fase de análise retrospetiva, reflete-se sobre as hipóteses adotadas. Estruturou-se um processo cíclico de (re)esquematização e desenvolvimento de métodos inovadores de ensino. Espera-se que a teoria de educação local resultante crie oportunidades para que professores, autores de manuais e investigadores possam adaptar os resultados às suas investigações ou prática letiva, tendo em consideração o contexto no qual trabalham.

Palavras chave. Construção de tarefas; teorias de instrução local; educação matemática; pesquisa de desenho.

Conception et recherché pour le development de théories de l'instruction locale

Résumé

L'innovation en didactique des mathématiques nécessite l'implication des professeurs, des auteurs de manuels, des législateurs et des chercheurs. Cet article propose une esquisse du rôle et de l'importance de la conception pédagogique orientée vers de nouvelles théories de l'instruction locale pour l'enseignement des mathématiques. Nous illustrons cette approche à travers une étude qui s'est donné pour but d'explorer comment les étudiants peuvent être soutenus dans le développement des principes fondamentaux des mathématiques du changement. Cette étude combine conception et recherche en trois phases successives. Dans la première phase, nous proposons la conception d'une trajectoire d'apprentissage hypothétique ainsi que d'activités pédagogiques; dans la phase d'expérimentation de l'enseignement, la trajectoire hypothétique est mise en œuvre ; enfin, dans la phase d'analyse a posteriori, les hypothèses avancées sont analysées. De la sorte se structure un processus cyclique de (re)conception et de développement d'enseignements innovants. La théorie de l'instruction locale qui en résulte devrait permettre aux enseignants, aux auteurs de manuels et aux chercheurs d'adapter les résultats à leurs recherches ou à leurs enseignements, en tenant compte du contexte dans lequel ils travaillent.

Paroles clés. Conception de tâches; théories de l'instruction locale; didactique des mathématiques; recherche orientée par la conception.

1. Introduction – why design is needed

One of the challenges in mathematics education research is trying to understand and improve teaching practices. In a traditional experimental research-design teaching program A is compared with teaching program B. But what to do when teaching program B is not available yet? Case studies and ethnographic studies offer better opportunities to describe characteristics of a certain teaching program in a specific context, but it may still be difficult to derive innovative characteristics of a teaching program. When we are interested in ways to change or innovate an educational situation for which no solution is at hand yet, something needs to be designed and the process of teaching and learning that is triggered by this design needs to be investigated, and in most cases redesigned. Such a cyclic process of design and research is referred to as design-based research. In design-based research theory development happens in interaction with experiments, experiments to understand and improve classroom situations (Bakker, Doorman & Drijvers, 2003). Characteristics of design-based research are that its aims are to develop theory, it is interventionist, it has a prospective and a reflective component in consecutive research cycles, and it does *real* work (ecological validity) (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003). The shift from a traditional experimental research-design to design-based research aligns with a shift from questioning what teaching program works to how and why a teaching program works.

One of the tasks for the mathematics education research community is to involve and support teachers in innovation-oriented processes. The resource for communicating an innovation-oriented idea with teachers that is at the core of this paper is a so-called local instruction theory. A local instruction theory embeds a sequence of activities for teaching a specific topic along with a theoretical underpinning of the most prominent characteristics of the sequence. This local instruction theory is expected to enable teachers to adapt tasks to the abilities and interests of students, while keeping the original end goals of a new and innovative approach (Gravemeijer, van Galen & Keijzer, 2005).

Design-based research in the context of this paper requires a process in which the design of instructional activities and teaching experiments are intertwined with the development and analysis of instructional theories for a specific topic. I will discuss a study on the introduction of the basic principles of calculus with the aim to illustrate the characteristics of such a design-based research approach. This approach tries to ensure a systematic process of design and analysis that offers opportunities to generalize findings over specific educational contexts. Therefore, the study has a dual goal:

- on the one hand, answering the question on the potential of a teaching and learning trajectory for the basic principles of calculus, and
- on the other, investigating and describing this potential in such a way that the results can be generalized and adopted by other teachers and researchers in other contexts.

Given these goals, design-based research or developmental research (Bakker, 2018; Gravemeijer, 1994, 1998) seems to be an appropriate research method. Cobb et al. (2003) refer to this type of research as performing design experiments, which they elucidate in the following manner:

> "Prototypically, design experiments entail both "engineering" particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them. This designed context is subject to test and revision, and successive iterations that result play a role similar to that of systematic variation in experiment." (Cobb et al., 2003, p. 9)

In this description, the two central aspects of this paper come to the fore in the design of means of support for particular forms of learning, also referred to as didactical engineering (Artigue, 2009), and the study of those forms of learning. In the study under discussion, the backbone of the design is formed by the development and revision of a hypothetical learning trajectory for the basic principles of calculus.

2. Design principles and design methodology

Design-based research has a cyclic character in which thought experiments and teaching experiments alternate. A cycle consists of three phases: preliminary design, teaching experiment, and retrospective analysis. A second feature of design-based research is the importance of the development of a learning trajectory made tangible in instructional activities. The design of instructional activities is more than a necessity for carrying out teaching experiments. The design process forces the researcher to make explicit choices, hypotheses and expectations that otherwise might remain implicit. The development of the design also indicates how the emphasis within the theoretical development may shift and how researcher's insights and hypotheses develop. As Edelson (2002) argues, design is a meaningful part of the research methodology:

> "(...) design research explicitly exploits the design process as an opportunity to advance the researchers understanding of teaching, learning, and educational systems. Design research may still incorporate the same types of outcome-based evaluation that characterise traditional theory testing, however, it recognizes design as an important approach to research in its own right." (p. 107)

This is particularly the case when the theoretical framework is under construction.

2.1. Phase 1: Design principles and HLT

The design of instructional activities in the study focused on in this paper includes the development of student text booklets and teacher guides. While designing these materials, choices and intentions were captured and motivated. When the materials were about to be finalised, these aims and expectations were described at the task level. Key items, that embodied the main phases in a hypothetical learning trajectory, were identified. These items reflected the relevant aspects of the intended learning process and were based on the conceptual analysis of the topic. The identification of key items guided observations and prepared for the retrospective data analysis. Finally, teacher guides as well as observation instructions were written, to make intentions and expectations clear to teachers and observers. During the design phase, products were presented to colleagues, teachers and observers. This led to feedback that in some cases forced the researcher to become more explicit about goals and aims, and that provided opportunities for improving all the materials.

While designing instructional activities, the key question is what meaningful problems may foster students' cognitive development according to the goals of the HLT. Three design principles guided the design process: guided reinvention, didactical phenomenology and emergent models.

The design principle of *guided reinvention* involves reconstructing a way of developing a mathematical concept from a given problem situation that gives the learners the impression that they (could) have invented the concept themselves (Freudenthal, 1991). A method for this can be to try to think how you would approach a problem situation if it were new to you. In practice, this is not always easy to do, because as a domain expert it is hard to think as if you were a freshman. Students' reasoning in contexts that are meaningful for them and the history of the domain can be informative on specific difficulties and potential starting points concerning concept development (e.g., Gravemeijer & Doorman, 1999).

The second design principle, *didactical phenomenology*, was developed by Hans Freudenthal. Didactical phenomenology aims at confronting the students with phenomena that "beg to be organised" by means of mathematical structures (Freudenthal, 1983, p. 32). In that way, students are invited to build up mathematical concepts. Meaningful contexts, from real life or "experientially real" in another way, are sources for generating such phenomena (De Lange, Burrill, Romberg & van Reeuwijk, 1993; Treffers, 1987). The question, therefore, is to find meaningful problem contexts that may foster the development of the targeted mathematical objects. The context should be perceived as meaningful and offer an orientation basis for mathematization.

The last remark leads to the third design principle, the use of *emergent models* (Gravemeijer, Cobb, Bowers & Whitenack, 2000; Van den Heuvel-Panhuizen, 2003). In the design phase we try to find problem situations that lead to models that initially represent the concrete problem situation, but in the meantime have the potential to develop into general models for an abstract world of mathematical objects and relations.

The expectations of the students' mental activities triggered by the classroom activities are elaborated in a hypothetical learning trajectory (HLT). The notion of a "hypothetical learning trajectory" is taken from Simon (1995). Originally, Simon used the HLT for designing and planning short teaching cycles of one or two lessons. In our study, however, a HLT is developed for teaching experiments that lasted for a longer sequence of lessons aiming at a concrete object of learning. As a consequence, the HLT comes close to the concept of a local instruction theory (Gravemeijer, 1994).

The development of an HLT involves the choice or design of instructional activities in relation to the assessment of the starting level of understanding, the formulation of the end goal and the conjectured mental activities of the students. Essential in Simon's notion of a HLT is that it is hypothetical; when the instructional activities are acted out, the teacher – or researcher in our case – will be looking for evidence of whether these *testable conjectures* can be verified, or should be rejected by what kind of o*bservation criteria.*

The concept of the HLT may seem to suggest that all students follow the same learning trajectory at the same speed. This is not how the HLT should be understood. Rather than a rigid structure, the HLT represents a learning route that is broader than one single track and has a certain bandwidth.

With an emphasis on the mental activities of the students and on the motivation of the expected results by the designer, the HLT concept is an adequate research instrument for monitoring the development of the designed instructional activities and the accompanying hypotheses. It provides a means of capturing the researcher's thinking and helps in getting from problem analysis to design solutions.

2.2. Teaching experiments

The second phase of the design research cycle is the phase of the teaching experiment, in which the prior expectations embedded in the HLT and the instructional activities are confronted with classroom reality. The term "teaching experiment" is borrowed from Steffe and Thompson (2000). The word "experiment" is not referring to an experimental group - control group design. In this section we explain how the teaching experiments were carried out; in particular, we pay attention to the data sampling techniques used during the teaching experiments.

The research questions share a process character: they concern the development of understanding of mathematical concepts. Therefore, we focussed on data that reflected the learning process and provided insight into the thinking of the students. The main sources of data were observations of student practice and interviews with students. The observations took place on two levels: classroom level and group level. Observations at classroom level concerned classroom discussions, explanations and demonstrations that were audio and video taped. These plenary observations were completed by written data from students, such as handed in tasks and notebooks. Observations at group level took place while the students were working on the instructional activities in pairs or small groups. Short interviews were held with pairs of students. In addition to this, the observers made field notes.

The lessons were evaluated with the teachers. In particular, the organisation of the next lesson and the content of the plenary parts were discussed. Also, decisions were taken about skipping (parts of) tasks because of time pressure. Such decisions were written down in the teaching experiment logbook.

2.3. Retrospective analysis

The third phase of a design research cycle is the phase of retrospective analysis. It includes data analysis, reflection on the findings and the formulation of the feed-forward for the next research cycle.

The first step of the retrospective analysis concerned *elaborating on the data*. A selection from video and audiotapes was made by event sampling. Criteria for the selection were the relevance of the fragment for the research questions and for the HLT of the teaching experiment in particular. Data concerning key items (see 2.1) was always selected and these selections were transcribed verbatim. The written work from the students was surveyed and analysed, especially the work on key items, tests and hand-in tasks. Results were summarised in partial analyses. This phase of the analysis consisted of *working through the protocols* with an open approach that was inspired by the constant comparative method (Glaser & Strauss, 1967; Strauss & Corbin, 1988). Remarkable events or trends were noted as conjectures and were confronted with the expectations based on the HLT and the instructional activities.

The second phase of analysis concerned *looking for trends* by means of sorting events and analysing patterns. The findings were summarised illustrated by prototypical observations. These conjectures were tested by surveying the data to find counterexamples or other interpretations, and by data triangulation: we analysed the other data sources, and in particular the written student material, to find instances that confirmed, rejected or refined the conjectures. Analysis of the written materials often evoked a reconsideration of the protocols. Analysis was continued in this way until saturation, which meant that no new elements were added to the analysis and no conclusions were subject to change.

The third phase in analysing the data was the *interpretation of the findings* and the comparison with the preliminary expectations of the HLT. Also, explanations for the differences between expectations and findings were developed. These conclusions and interpretations functioned as feed-forward for the formulation of new hypotheses for the next cycle in the research.

3. Background and design of HLT

The aim of this study is to find out how students can learn the basic principles of calculus and kinematics by modelling motion. Nowadays, graphs are used in calculus and kinematics education as representations for describing change of velocity or distance travelled during a time interval. Students are expected to give meaning to the relation between distance travelled and velocity through characteristics of these graphs such as area and slope. The use of such instructional materials is based on a representational view (Cobb, Yackel & Wood, 1992), which assumes that instructional materials can represent scientific knowledge, and that scientific concepts can be made accessible without fully taking into account the limitations of the knowledgebase of the students into which they have to be integrated.

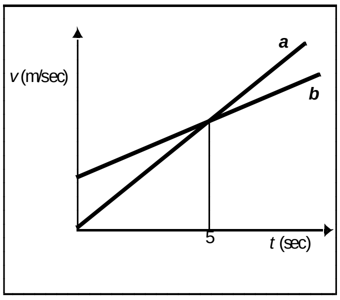


Figure 1. The change in velocity of car *a* and car *b*

What we tried to prevent with this learning route was a kind of reasoning that we observed in physics lessons. For instance, when analysing a graph depicting the change in velocity of two cars *a* and *b* (see Figure 1), most students answered that at the point of intersection car *a* overtakes car *b*. This pictorial interpretation of what happens at the point of intersection, one line passing the other, dominates the thinking of the students, and they find it hard to deduce that the only thing you know is that after 5 seconds car *a* and *b* have exactly the same speed.

Such interpretations might be caused by a process in which students are not sufficiently involved in the construction process of such graphs of motion. The continuous graphs are presented to students as if they are self-explanatory. In contrast, our learning route is inspired by the potential of students' graphical inventions when asked to represent motion. One of the reported examples concerns the story of a car decreasing speed, stopping, and increasing speed. The graphical solutions of the students entailed a rich variety of discrete graphs involving dots and dashes representing decreasing and increasing intervals (DiSessa, Hammer, Shern & Kolpakowski, 1991). Furthermore, the domain history confirmed design choices related to the use of discrete graphs to provide students with meaningful tools that afford them a way to reason with patterns in differences and taking sums before working with continuous graphs that need formalization with limits (Doorman & Van Maanen, 2008).

The initial task sequence on the basic principles of calculus addresses the so-called discrete case of the main theorem of calculus. In this route, the creation, use and adaptation of various graphical representations are interwoven with learners' activities in a series of science-practices, from modelling discrete measurements to reasoning with continuous models of motion. The trajectory starts with questions about a weather forecast. The teacher discusses the change of position of a hurricane with students: When will it reach land? (see Figure 2).

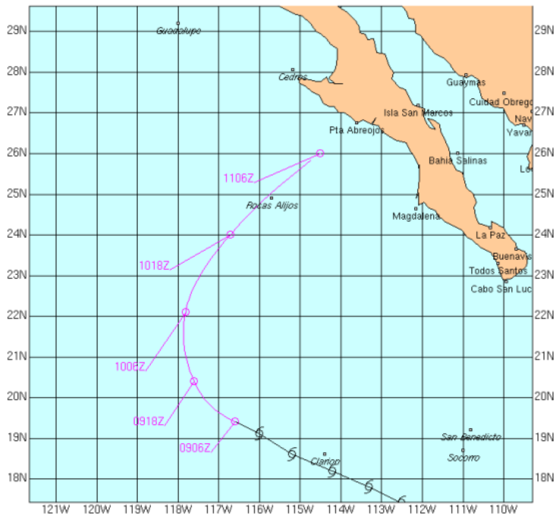


Figure 2. Predicted successive positions of a hurricane (12h between positions)

This problem is posed as a leading question throughout the unit as a context for the need of grasping change. After the emergence of time series as useful tools for describing change of position, students work with situations that are described by stroboscopic photographs. The idea is that students come up with measurements of displacements, and that it makes sense to display them graphically for finding and extrapolating patterns. Two types of discrete graphs are discussed, namely, interval graphs (distances between successive positions) and graphs of the total distance travelled. Note that discrete graphs are not introduced as an arbitrary symbol system, but emerge as models of discrete approximations of a motion, that link up with prior activities and students' experiences. By using the computer program Flash[1] students are able to investigate many situations. During these activities the intention was to shift their attention from describing specific situations to properties of these discrete graphs and the relation with kinematical concepts. During the first teaching cycle students appeared to have difficulties with interpreting and using the graphs provided by the computer tool. Consequently, opportunities for students to produce ways of graphing motion before they use the tool and guidelines for the teacher how to use students' ideas for introducing the tools were added.

We focus on students' contributions during the activities, and how we can build upon their contributions towards the intended attainment targets. Consequently, for observing and analysing the development in their reasoning we use the design-based research approach of planning and testing the envisioned trajectory in classroom situations. We are interested in *how* the trajectory works and can be improved, instead of trying to decide *whether* it works (see Table 1).

The sequence is tried out and revised during teaching experiments in three tenth-grade classes. We collected data by video and audio taping whole class discussions and group work. The videotapes were used to analyse students' oral discourses and written materials with respect to the conjectured teaching and learning process.

Table 1. *From hypotheses to observation criteria*

Testable conjectures	*Observation criteria*
Do students perceive the problem situations as intended, and fits their reasoning the intended development from trace graphs?	In their initial (intuitive) reasoning about the weather problems, students refer to the intervals between successive positions and relate changing lengths of intervals with velocity. Students invent ways to describe and investigate patterns in intervals.
Does the previously planned sequence of graphical tools fit students' thinking and foster advanced reasoning?	The way students reason with the graphs changes from context-oriented (referring to intervals in the stroboscopic pictures) to an orientation on features of and relations between interval graphs and graphs of total distances travelled.
Do the representations in the computer tool fit prior reasoning and afford advanced reasoning?	Initially, students use the stroboscopic pictures and prior activities to signify the graphs. During their work, students increasingly use the graphs offered for solving the posed problems. As a consequence, they simultaneously invent use of and relations between these tools.

Teaching experiment phase

We illustrate the change in how students think and talk about a model with an episode describing students' work with the computer program Flash. Initially, while they are working with Flash, students refer to distances between successive positions, and later on they start reasoning with the global shapes of graphs and their relationships with the represented motion. An example of such reasoning concerns a task about a zebra that is running at constant speed and a cheetah that starts hunting the zebra. The question is whether the cheetah will catch up with the zebra. In the graphs (see Figure 3) the successive measurements of the zebra and the cheetah are displayed. In the graph on the right can be seen that the zebra (blue) is covering constant distances in equal time

[1] http://www.fi.uu.nl/toepassingen/00197/flits_en/balletje.html

intervals 1, 2, 3, …, while the distances covered by the cheetah (red) initially increase and from time interval 6 decrease. The following discussion takes place between an observer and two students (Rob and Anna).

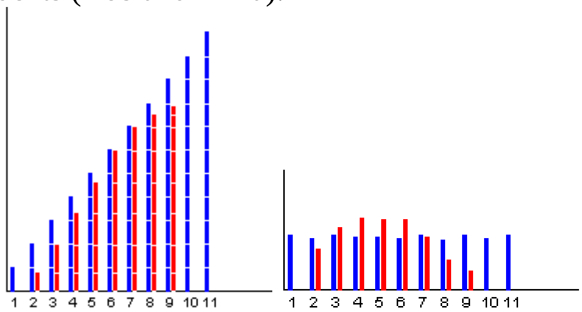


Figure 3. Distance-travelled and interval graphs in Flash

Observer	*Oh, yes. So why did you choose the one for the total distance* [left graph]?
Rob	*Because it's the total distance that they cover and then you can-*
Anna	*Then you can see if they catch up with each other.*
Observer	*And can't you see that in the other* [right graph]? *There you can also see that the red catches up with blue?*
Rob	*Yes, but -*
Anna	*Yes, but that's at one moment. That only means that it's going faster at that moment but not that it'll catch up with the zebra.*

Retrospective analysis

A difference between the interval graph and the distance-travelled graph is the difference between the interpretations of the horizontal (time) axis. A value in the distance-travelled graph represents a distance from the start until the corresponding time, while a value in the interval graph represents a distance in the corresponding time interval. Anna's last observation is an important step in the process of building the model of a velocity-time graph (and everything that comes with it).

The qualitative analyses show that during the practices students re-invent and develop graphical symbolisations, as well as the language and the scientific concepts that come with them. However, these inventions only became explicit after interventions by an observer or by the teacher. We found that the teacher had a crucial role during the classroom discussions. It was not always easy to organise the discussions in line with the intended process. Sometimes the teacher reacted to students' contributions in terms of the representational inscriptions or concepts aimed at (Meira, 1995; Roth & McGinn, 1998). In those cases, students awaited further explanation. The discussions appeared to be especially productive when the teacher organised classroom discussions about students' contributions in such a way that the students themselves posed the problems that had to be solved, and reflected on their answers. In a second teaching experiment we arranged a setting where the teacher had more information about the possible contributions of the students and the way in which they could be organised. Additionally, we designed activities for classroom discussions.

The HLT for the second teaching experiment is summarized in Figure 4. This summary shows how the graphical tools emerge in relation to the task sequence. The (discrete) graphs become meaningful tools for the students through their activity with analysing patterns in successive intervals in time series and trace graphs. The activity on one level creates the need and becomes the signified (the 'imagery') for an activity with a new, more advanced tool on the next level (Cobb, Gravemeijer, Yackel, McClain

& Whitenack, 1997; Van Oers, 2000). Furthermore, the figure illustrates the interaction between the development of these tools and the development of meaning related to the mathematical concepts of change.

In this approach the construction and interpretation of graphs, trace graphs as well as the 2-dimensional discrete graphs, and the scientific concepts are rooted in the activities of the students. This process has the potential to ensure that the mathematical and physical concepts aimed at are connected to students' understanding of everyday phenomena. On the basis of our findings we conclude that classroom discussions where students discuss their solutions and pose new problems to be solved, are essential for a learning process during which symbolisations and knowledge of motion co-evolve.

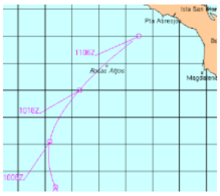
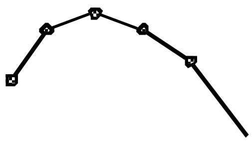
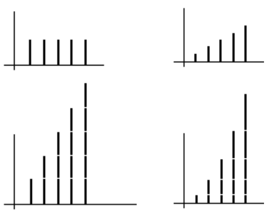
<table>
<tr><th>Tool</th><th>Imagery</th><th>Activity</th><th>Concepts</th></tr>
<tr><td rowspan="2">
time series (e.g. satellite photos, stroboscopic pictures)</td><td rowspan="2">real world representations signify real world situations</td><td>predicting motion (e.g. in the context of weather predictions)</td><td>displacements in equal time intervals as an aid for describing and predicting change</td></tr>
<tr><td colspan="2">should result in a feeling that the ability to predict motion with discrete data is an important issue</td></tr>
<tr><td rowspan="2">trace graphs of successive locations
</td><td rowspan="2">signifies a series of successive displacements in equal time intervals</td><td>compare, look for patterns in displacements and make predictions by extrapolating these patterns</td><td>displacements as a measure of speed, of changing positions, but difficult to extrapolate</td></tr>
<tr><td colspan="2">resulting in a willingness to find other ways to display intervals for viewing and extrapolating patterns in them</td></tr>
<tr><td rowspan="2">discrete 2-dim graphs
</td><td rowspan="2">signifies patterns in trace graphs
(and cumulative)</td><td>compare patterns and use graphs for reasoning and making predictions about motion (also at certain moments: interpolate graphs)
refine your measurements for a better prediction: intervals decrease</td><td>intervals depicting patterns in motion;
linear line of summit in graph of displacements or graph of distances traveled;
problems with predictions of instantaneous velocity</td></tr>
<tr><td colspan="2">should result in the need to know more about the relation between sums and differences, and in the need to know how to determine and depict velocity</td></tr>
</table>

Figure 4. The HLT for the second teaching experiment

The learning process aimed at asked for a careful design of teaching-learning trajectories involving an intertwined process of symbol introduction and meaning making. During that process, students get opportunities for creating their own constructions and reflecting on them. Realistic contexts proved important in that.

4. Discussion

The case study illustrated how an innovative approach for a topic in mathematics can be investigated. The use of semiotic theories turned out useful for planning and analysing a process of symbolising and development of meaning. The Hypothetical Learning Trajectory appeared to be a useful instrument in all phases of this design-based research. During the design phase it is the theoretically grounded vision of the learning process, which is specified for concrete instructional activities. During the teaching experiments, the HLT offers a framework for decisions during the teaching experiment and guides observations and data collections. In the retrospective analysis phase, the HLT serves as a guideline for data selection and offered conjectures that could be tested during the analysis. The final HLT is a reconstruction of a sequence of concepts, tools and instructional activities, which constitute the effective elements of a learning trajectory. In this manner, the result is a well-considered and empirically grounded local instruction theory for the basic principles of calculus. The HLT, together with a description of the cyclic process of design and research, enables others to retrace the learning process of the research team. Understanding the how and why of the specified steps makes it possible to let that learning process become your own and to adapt findings to your own context.

When rigorous designs are implemented in everyday classroom practice, in many cases original designs go through a certain mutation which probably leads to adaptations which are not under theoretical control (Artigue, 2009). Nevertheless, the local instruction theory describes and underpins how the intervention is expected to work and helps teachers and instructional designers to adjust and adapt the instructional activities in the spirit of the original intentions (De Beer, Gravemeijer & van Eijck, 2018).

The calculus-example in this paper illustrates design-based research as a systematic approach for innovation-oriented studies. The close connections between design and theory development is expected to offer teachers and researchers opportunities to translate the results to their own teaching or researching practice. Task design is crucial in this process for translating theoretical ideas into classroom practices and for communicating the main idea among the different communities.

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Design and research for developing local instruction theories

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Extended abstract

Innovation in mathematics education needs the involvement of teachers, textbook authors, policy makers and researchers. One of the challenges in innovation processes is the communication of a key idea among these different communities. Carefully designed tasks can afford communication as they have the potential to illustrate a theoretical idea and a classroom situation in which this task is enacted. This paper sketches the role and importance of instructional design aiming at new and innovative local instruction theories in mathematics education. The approach is illustrated with a study that investigated how students can be involved in the development of the basic principles of the mathematics of change. The study combines design and research in three successive phases. In the first phase instructional activities are designed based upon prior educational research and an historical exploration of the topic giving rise to the importance of discrete models in reasoning about change. One of the example tasks is cast within the context of a storyline about a hurricane approaching a coastline. Successive positions are given and the pattern between these positions can be extrapolated to predict when and where the hurricane will hit the coast. These activities are elaborated in a hypothetical learning trajectory that also includes expectations and observation criteria about the teaching and learning process. One of the hypotheses is that connecting patterns in changing intervals and patterns in total distances travelled will support students in developing an understanding of the basic principles of the mathematics of change. For example, intervals with a constant length result in a linear growth of distance travelled. Furthermore, a computer tool is expected to be helpful in supporting students' further exploration of connections between interval graphs and distance travelled graphs. In the following teaching experiment phase the trajectory is acted out and data on the actual process are collected. In the phase of the retrospective analysis is reflected upon the articulated hypotheses, for instance about the role of the hurricane context and the role of the discrete models, by comparing the hypotheses with the available data. This approach structures a cyclic process of design and redesign that, ideally, is converging into an empirically supported local instruction theory. We argue that such an instruction theory together with tasks and illustrative classroom vignettes help to communicate the envisioned process of teaching and learning and can create opportunities for teachers, textbook authors and researchers to take contextual factors into account and to adapt the results for their own research or teaching practice.